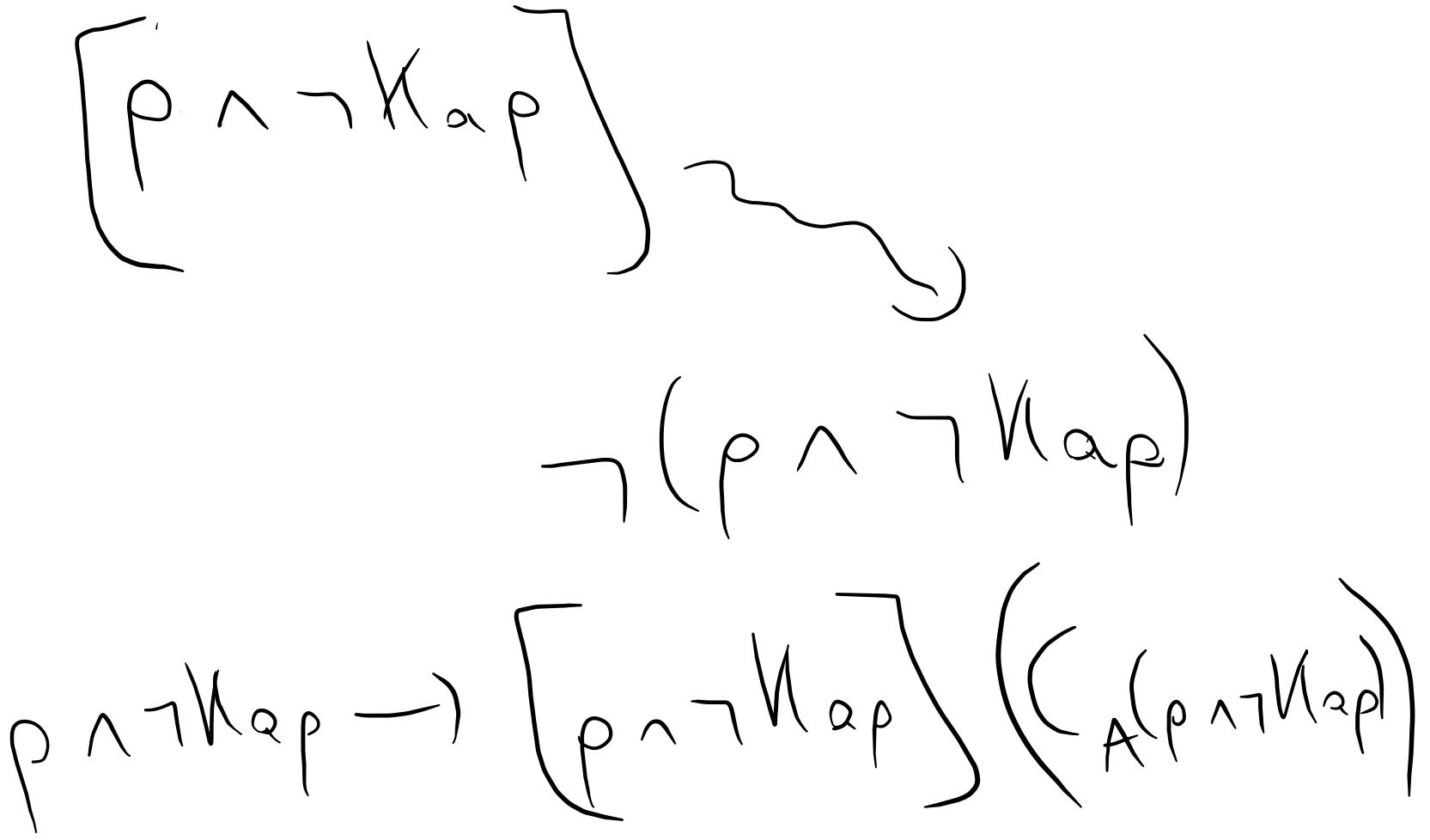
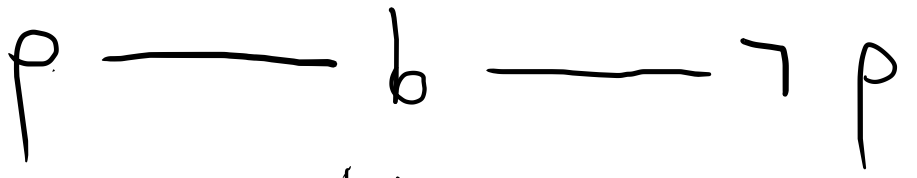
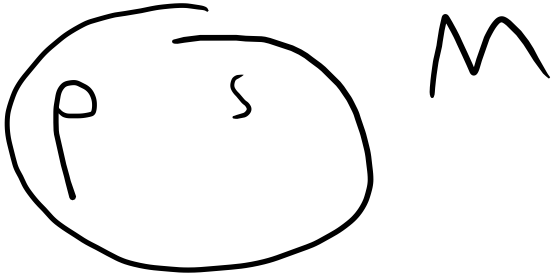

[ρ ∧ ¬K_a ρ]
¬(ρ ∧ ¬K_a ρ)
ρ ∧ ¬K_a ρ → [ρ ∧ ¬K_a ρ](C_A(ρ ∧ ¬K_a ρ)

1 0 b : Bill



a : Anne

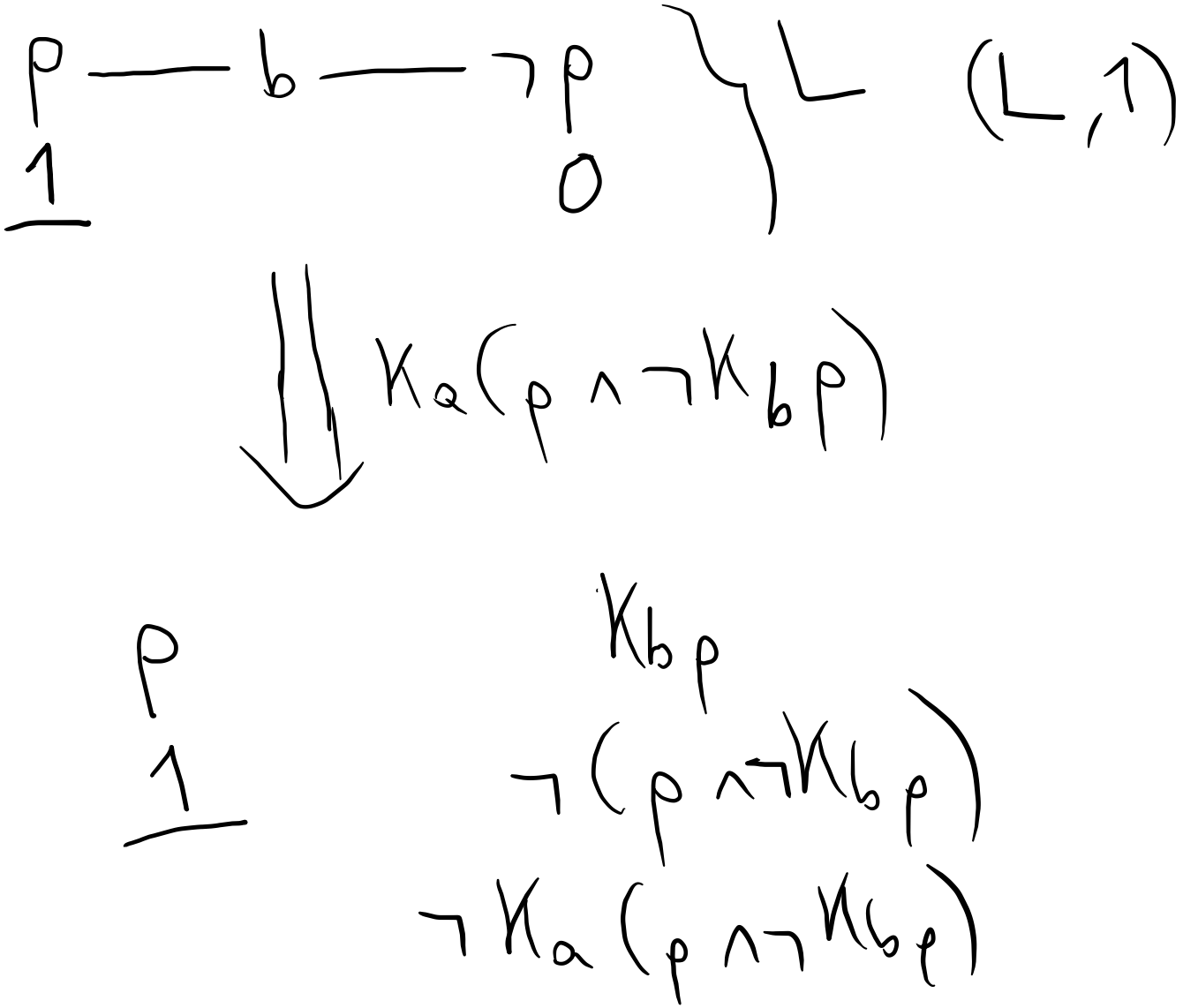


M

$M, s \models K_a p$

$M, s \models C_{ab} K_a p$

$M, s \models K_a p \rightarrow C_{ab} K_a p$


p — b — ¬p
1
0
L
(L, 1)
$K_a(p \wedge \neg K_b p)$
p
1
$K_b p$
$\neg(p \wedge \neg K_b p)$
$\neg K_a(p \wedge \neg K_b p)$

$M, s \models \langle \phi \rangle \psi \iff$

$M, s \models \phi$ and $M|_{\phi}, s \models \psi$

$M, s \models [\phi] \psi \iff$

if $M, s \models \phi$ then $M|_{\phi}, s \models \psi$

ϕ is successful iff

$\langle \phi \rangle \phi$ is valid iff

$(\forall M)(\forall s)\left[M, s \models \phi \text{ and } M|_{\phi}, s \models \phi \right]$

ϕ is successfull
ψ is successfull

but $\phi \wedge \psi$ is not successfull

$\phi = p \quad \psi = \neg K_a p \quad \phi \wedge \psi = p \wedge \neg K_a p$

$\models [\neg K_a p] \neg K_a p$ Let M, s be arbitrary

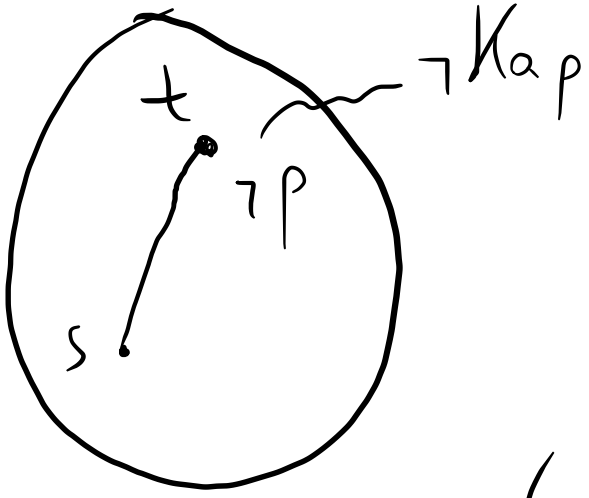
$M, s \models [\neg K_a p] \neg K_a p \iff$

$(M, s \models \neg K_a p \Rightarrow M|_{\neg K_a p}, s \models \neg K_a p)$

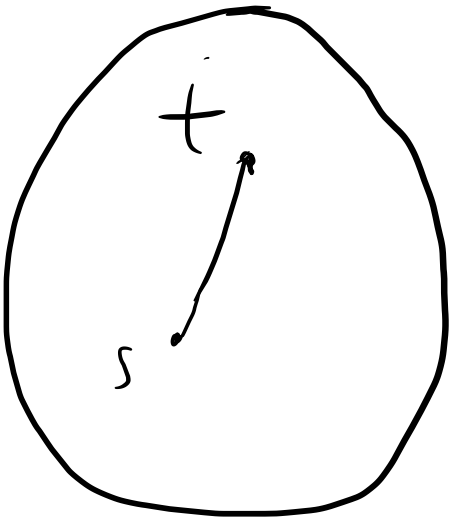
$M' = M|_{\neg K_a p}$

$M, s \not\models \neg K_a p$

M



$M' = M|_{\neg K_a p}$



$M', s \models \neg K_a p$

ϕ is successfull, but $\neg\phi$ is unsuccessful

$\phi = p \to K_a p \qquad \neg\phi = p \wedge \neg K_a p$

$\models [\phi]\phi, \quad M, s \models [\phi]\phi \iff$

$\left(M, s \models (p \to K_a p) \right) \Rightarrow M|_{p \to K_a p}, s \models p \to K_a p$

$M' = M|_{p \to Kap}$ $\quad M, s \not\models p \to Kap$ (*)

if $M', s \not\models p$ then $M', s \models Kap$

$\Downarrow$ (*)

$M, s \not\models p \implies M, s \models Kap$

ϕ and ψ successful, but $\phi \to \psi$ is unsuccessful

$$\phi = \neg(p \wedge \neg K_a p) = p \to K_a p$$

$$\psi = p \vee \neg p$$

$$\phi \to \psi \equiv \neg\phi \vee \top \equiv \neg\phi \equiv$$

$$\neg(p \to K_a p) =$$

$$p \wedge \neg K_a p$$

ϕ and ψ is successful, but $[\phi]\psi$ is not successful

$\phi = p \to K_a p = \neg p \vee K_a p \not\models [[\phi]\psi][\phi]\psi$

$\psi = \neg K_a p$

$M, s \models [\phi]\psi$

$\phi \in L_{KC(\square)}(A,P)$ then $C_A\phi$ is successful, i.e. $[C_A\phi]C_A\phi$ is valid

---

$M,s \models [C_A\phi]C_A\phi \iff$

$M,s \models C_A\phi \Rightarrow M|_{C_A\phi}, s \models C_A\phi$

$$\sim_A = \bigcup_{\alpha \in A} \sim_\alpha$$

$M, s \models C_A$.

$M, s \models C_A\phi$

$\Downarrow$

$M|_{[s]_{\sim A}}, s \models C_A\phi$

$\Downarrow$

$M|_{[\![C_A\phi]\!]}, s \models C_A\phi$

$\Downarrow$

$M|_{C_A\phi}, s \models C_A\phi$

$p \xrightarrow{1} \bullet \xrightarrow{b} \bullet \xrightarrow{} p$ $\quad L, 1 \quad A = \{a, b\}$

$L, 1 \not\models [C_a(p \wedge \neg K_b p)]\, C_a(p \wedge \neg K_b p)$

$C_a(\varphi \wedge \neg K_a p)$

1 $\quad L', 1 \not\models C_a(p \wedge \neg K_b p)$