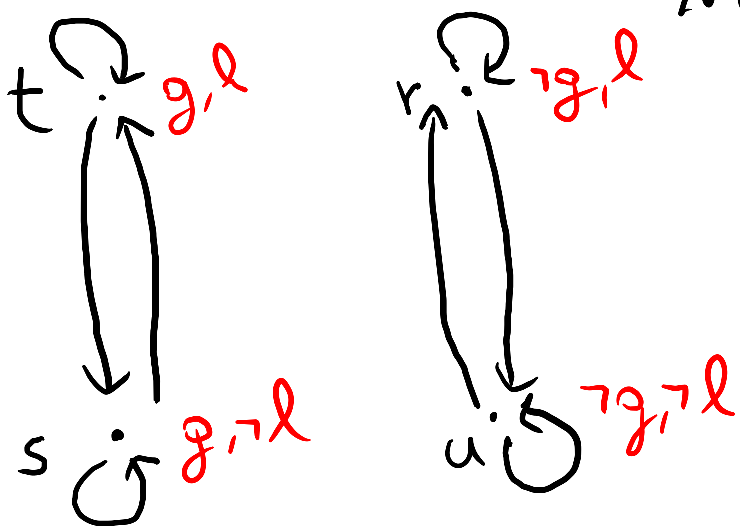
Exercise 2.8 (DEL)

g: it is sunny in Groningen l: it is sunny in Liverpool



$M = \langle S, R, V \rangle$

$S = \{r, s, t, u\}$

$V(g) = \{t, u\}$

$V(l) = \{t, r\}$

one agent that lives in Groningen

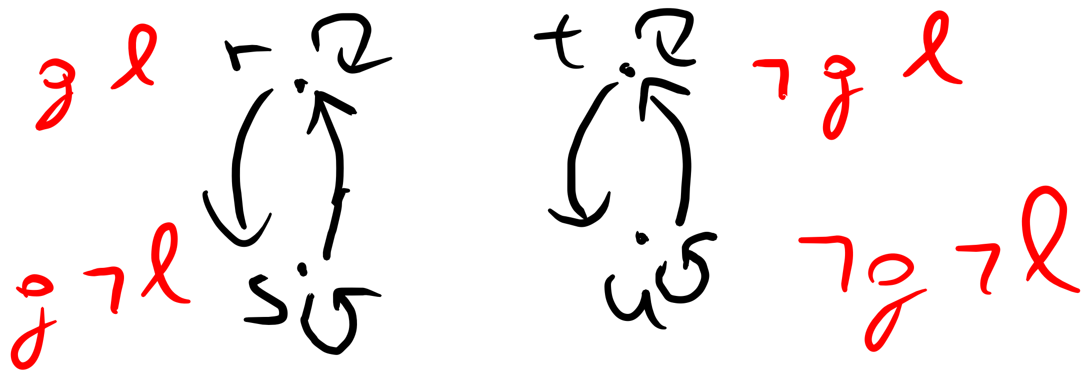
$M, t \models \Diamond g \wedge \Diamond l \wedge \Diamond \neg l \iff$

$M, t \models \Diamond g$ and $M, t \models \Diamond l$ and $M, t \models \Diamond \neg l \iff$

$(\exists w_1)(tRw_1$ and $M, w_1 \models g)$ and $(\exists w_2)(tRw_2$ and $M, w_2 \models l)$ and

$(\exists w_3)(tRw_3$ and $M, w_3 \models \neg l)$

$M, r \models \Diamond \neg g \wedge \neg \Box l$



$* \quad \neg \Box \ell \iff \Diamond \neg \ell$

$M,r \models \Box \ell \iff (\forall w)[rRw \Rightarrow M,w \models \ell]$

it does not hold!!

$M,r \models \Box(\Box g \wedge \neg \Box \ell) \overset{*}{\iff}$

$(\forall w)[rRw \Rightarrow (M,w \models \Box g \wedge \Diamond \neg \ell)] \iff$

$M,r \models \Box g$ and $M,r \models \Diamond \neg \ell$ and $M,s \models \Box g$ and $M,s \models \Diamond \neg \ell$

true due to r, s

true due to s

true due to r, s

true due to s

Validity

- A formula ϕ is called valid in a model $M = (S, V, R)$ if for every $w \in S$ it holds $M, w \models \phi$ (we write $M \models \phi$.)
- A formula ϕ is called valid if for every model M, it holds $M \models \phi$. In this case we write $\models \phi$

Examples

a) $\models \Box(\phi \vee \neg\phi)$. Indeed, let $M = \langle S, V, R \rangle$ and $w \in S$. We have $M, w \models \Box(\phi \vee \neg\phi) \iff$
$(\forall u \in S)[wRu \Rightarrow M, u \models \phi \vee \neg\phi]$

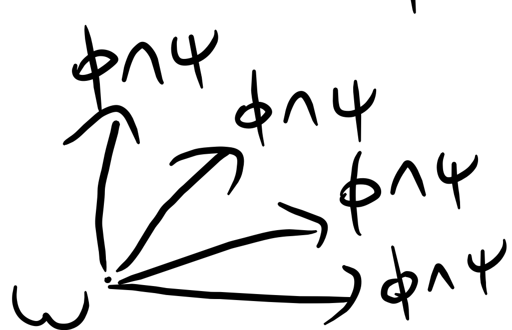
b) $\models \Box(\phi \wedge \psi) \rightarrow (\Box\phi \wedge \Box\psi)$. Let $M = \langle S, V, R \rangle$ and $w \in S$. We have $M, w \models \Box(\phi \wedge \psi) \rightarrow (\Box\phi \wedge \Box\psi)$

We assume $M, w \models \Box(\phi \wedge \psi) \Rightarrow (\forall u)[wRu \Rightarrow M, u \models \phi \wedge \psi] \Rightarrow$
$(\forall u)[wRu \Rightarrow (M, u \models \phi \text{ and } M, u \models \psi)]$ (*)

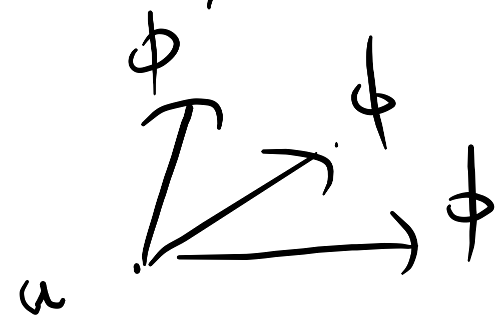
$M, w \models \Box\phi \wedge \Box\psi \iff M, w \models \Box\phi$ and $M, w \models \Box\psi \iff$

$(\forall u_1)(wRu_1 \Rightarrow M, u_1 \models \phi)$ and $(\forall u_2)(wRu_2 \Rightarrow M, u_2 \models \psi]$
which is true due to (*)

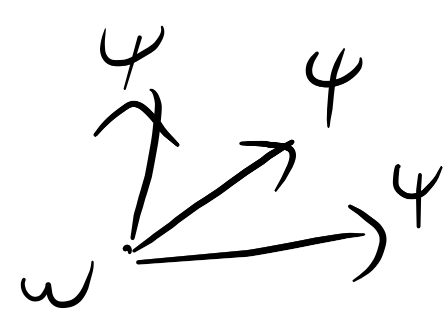
$M, w \models \Box(\phi \wedge \psi)$



$M, w \models \Box\phi$



$M, w \models \Box\psi$



$\gamma)$ $\models \Diamond(\phi \vee \psi) \to (\Diamond\phi \vee \Diamond\psi)$

Let $M = \langle S, R, V \rangle$ and $w \in S$. We have $M, w \models \Diamond(\phi \vee \psi) \to (\Diamond\phi \vee \Diamond\psi)$. So let

$M, w \models \Diamond(\phi \vee \psi) \Rightarrow (\exists u)[wRu \text{ and } M, u \models \phi \vee \psi]$

$(\exists u)[wRu \text{ and } (M, u \models \phi \text{ or } M, u \models \psi)] \Rightarrow$

$(\exists u)[(wRu \text{ and } M, u \models \phi) \text{ or } (wRu \text{ and } M, u \models \psi)]$

$(\exists u)[wRu \text{ and } M, u \models \phi] \text{ or } (\exists u)[wRu \text{ and } M, u \models \psi]$

$M, w \models \Diamond\phi$ or $M, w \models \Diamond\psi$

$M, w \models \Diamond\phi \vee \Diamond\psi$

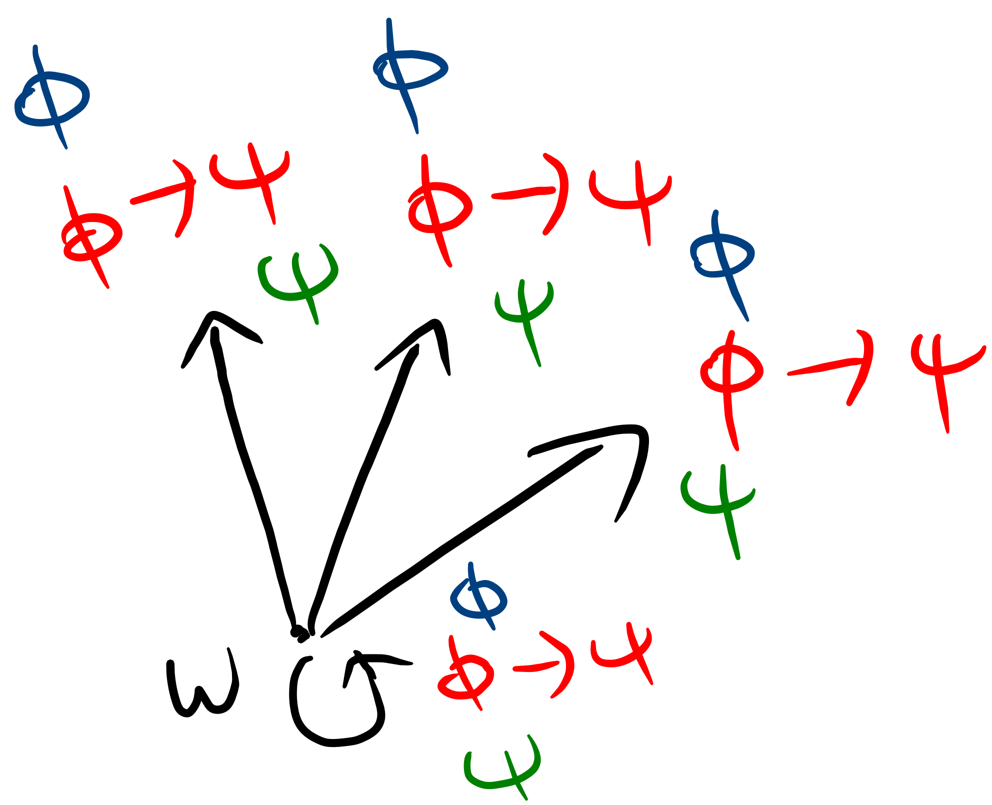
8) $\models \underbrace{\Box(\phi \to \psi)}_{1} \wedge \underbrace{\Box \phi}_{2} \to \underbrace{\Box \psi}_{3}$

$M = \langle S, R, V \rangle \quad w \in S$

$M, w \models \Box(\phi \to \psi) \wedge \Box \phi \Rightarrow$

$(\forall u_1)[wRu_1 \Rightarrow (M, u_1 \models \phi \to \psi$ and $M, u_1 \models \phi)] \Rightarrow$

$(\forall u_1)[wRu_1 \Rightarrow M, u_1 \models \psi] \Rightarrow M, w \models \Box \phi$

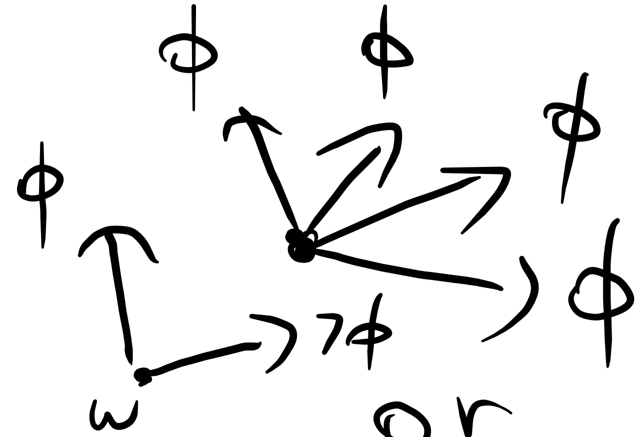


$\Diamond(\phi \to \psi) \wedge \Diamond \phi \cancel{\to} \Diamond \psi$

$\Box(\phi \to \psi) \wedge \Diamond \phi \to \Diamond \psi$

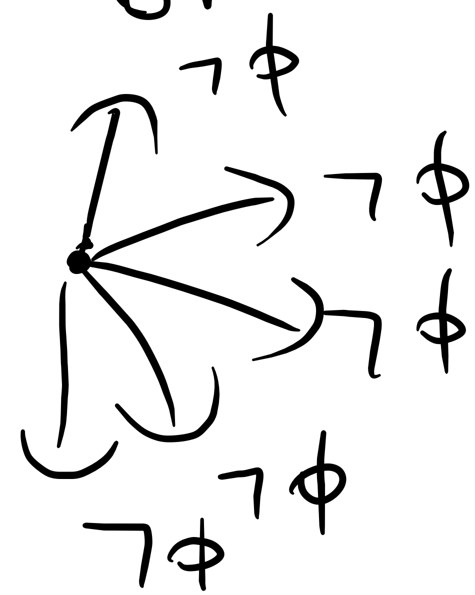
$\Diamond(\phi \to \psi) \wedge \Box \phi \to \Diamond \psi$

ε) $\models^{?} \Box\phi \vee \Box\neg\phi$



It suffices to find an $M=\langle S,R,V\rangle$ and a $w\in S$ such

or



$M,w\models\neg(\Box\phi\vee\Box\neg\phi) \Rightarrow$

$M,w\models\neg\Box\phi\wedge\neg\Box\neg\phi \Rightarrow$

$M,w\models\Diamond\neg\phi\wedge\Diamond\phi \Rightarrow$

$M,w\models\Diamond\neg\phi$ and $M,w\models\Diamond\phi$

$(\exists u_1)[wRu_1 \Rightarrow M,u_1\models\neg\phi]$ and

$(\exists u_2)[wRu_2 \Rightarrow M,u_2\models\phi]$

$\neg\forall x\phi \Leftrightarrow \exists x\neg\phi$

$\neg\Box\phi \Leftrightarrow \Diamond\neg\phi$

$\phi=p \quad M'=\langle S',R',V'\rangle$

$S'=\{w,u_1,u_2\}$

$R'=\{(w,u_1),(w,u_2)\}$

$V(p)=\{u_2\}$