# GAMES, DYNAMICS & LEARNING

**Panayotis Mertikopoulos**[1]

joint with

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[1]French National Center for Scientific Research (CNRS) & Criteo AI Lab

[2]NTUA

[3]Columbia University

ECE-NTUA – May 14, 2021

# GAMES, DYNAMICS & LEARNING

## BACKGROUND & MOTIVATION

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## Outline

## Game 1: Congestion models

**Planning your commute:** not sure when to leave, nor who will be on the road

**Figure:** A game with a random set of players

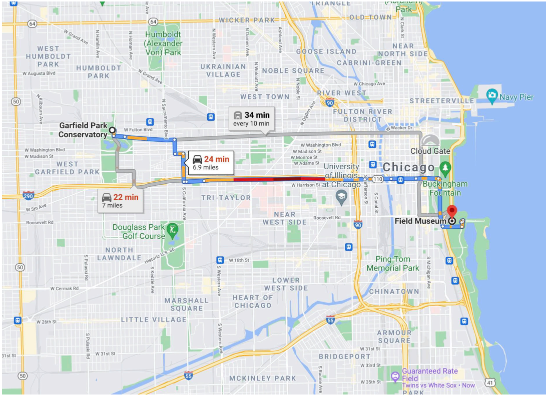
## *The price of congestion*

In the US alone, congestion cost $305 billion in 2017 (≈1.6% of GDP)

[source: INRIX]

- Lost productivity
- Fuel waste
- Environmental impact, quality of life,…

## *Game of roads*




### The city of Chicago

- 2,700,000 people
- 1,261,000 daily trips
- 933 nodes
- 2950 edges
- 870,000 o/d pairs
- $\approx 2 * 10^{16}$ paths

**A very large game!**

## Outline

Commuting

Machine learning

Course outline

## Game 2: A graphical Turing test

Which person is real?

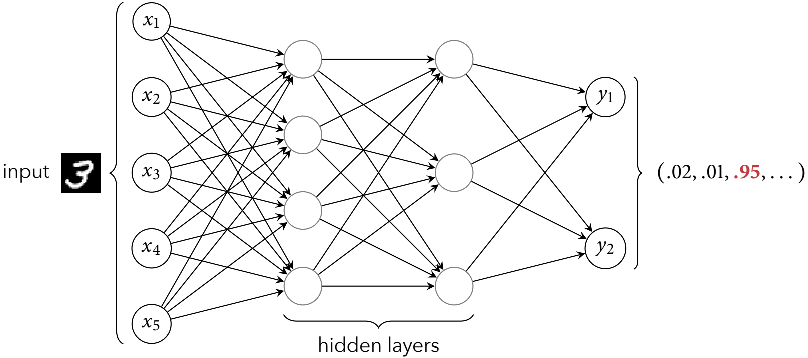
## Game 2: A graphical Turing test

Which person is real?

[Spoiler: https://thispersondoesnotexist.com]

## Neural networks

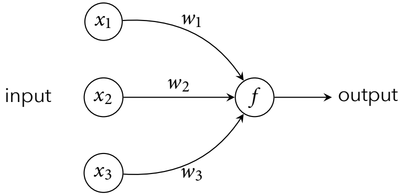
The workhorse of deep learning:




The deep learning revolution: breaking the human perception barrier (2010's)

## Neurons

The atoms of any deep learning architecture are its **neurons:**

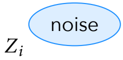



- **Input** could be binary $\{0,1\}$ or real (e.g., average intensity of image)
- Inputs weighed with **weight coefficients** $w_i$
- Neuron **activates** on value of $f(\sum_i w_i x_i)$

### Examples

1. *Perceptron:* binary inputs, step function activation
2. *Sigmoid neuron:* real inputs, $\tanh$ activation
3. *ReLU:* real inputs, rectified linear activation ($f(z) = [z]_+$)

## The schematics of GANs



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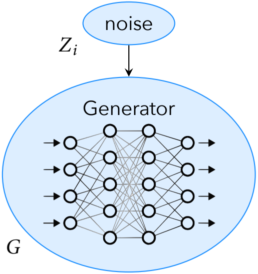

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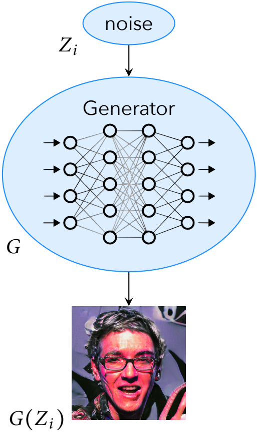

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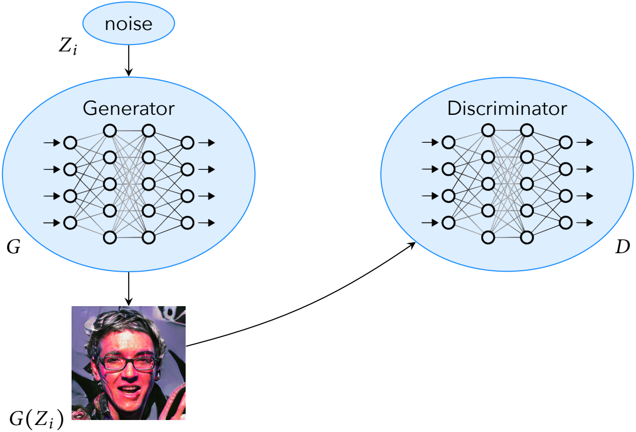

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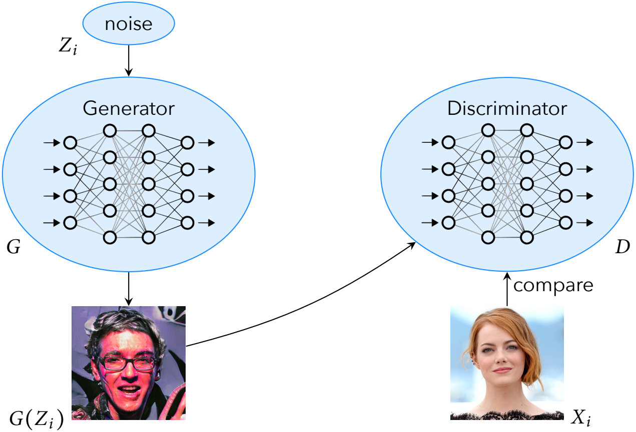
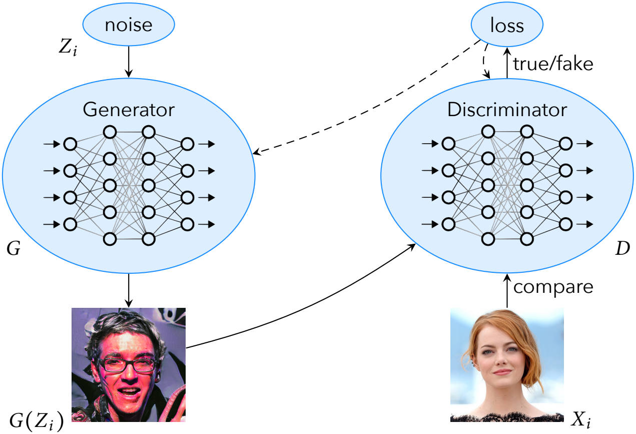


## *The schematics of GANs*




**Model likelihood:** $L(G,D) = \prod_{i=1}^{N} D(X_i) \times \prod_{i=1}^{N} (1 - D(G(Z_i)))$

## GAN training

How to find good generators ($G$) and discriminators ($D$)?

**Discriminator:** maximize (log-)likelihood estimation

$$\max_{D \in \mathcal{D}} \log L(G, D)$$

**Generator:** minimize the resulting divergence

$$\min_{G \in \mathcal{G}} \max_{D \in \mathcal{D}} \log L(G, D)$$

**A very complex zero-sum game!**

## FailGAN

The face of failure in GANs:

[A StyleGAN after 8 days of training at Nvidia headquarters (!!!)]

# Outline

## *Many questions*

1. **How should we model player interactions?**
   - Urban traffic ≠ transit systems ≠ packet networks ≠ …
   - Rational agents ≠ humans ≠ AI algorithms ≠ …
   - Competition ≠ congestion ≠ coordination ≠ …
2. **What is a desired operational state?**
   - Social optimum ≠ equilibrium ≠ …
   - Static (equilibrium, social optimum) ≠ Bayesian ≠ online (regret) ≠ …
3. **How to compute it?**
   - Calculation ≠ learning ≠ implementation
   - Informational constraints: feedback, bounded rationality, uncertainty, …

**No single answer**

# Lecture plan

1. Part 1: Basic concepts
    - What's in a game?
    - Nash equilibrium
    - Other notions of rationality
2. Part 2: Game dynamics
    - Basic definitions
    - The replicator dynamics
    - Rationality analysis
3. Part 3: Learning in finite games
    - Regret
    - No-regret learning: dynamics and algorithms
    - Equilibrium convergence properties
4. Part 4: Learning in continuous games
    - Online convex optimization
    - Algorithms and guarantees
    - Equilibrium convergence properties

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