Action Models

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Dynamic Epistemic Logic: Action Models,

by van Ditmarsch, Hans, van der Hoek, Wiebe, Kooi, Barteld

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Convey the same information for all agents

- Restriction of the model
- Restriction of the accessibility relations

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Epistemic Actions

Convey different information to different agents

- Refinement of accessibility relations

Basic Example

Buy or Sell?

Two stockbrokers Anne and Bill, having a little break in a Wall Street bar, sitting at a table. A messenger comes in and delivers a letter to Anne. On the envelope is written "urgently requested data on United Agents". Anne opens and reads the letter in the presence of Bill.

(United Agents is doing well.)

Basic Example Analysis (1/3)

Modelled as an epistemic state, with agents Anne (α) and Bill (b):

- Atom p: United Agents is doing well
- Atom $\neg p$: United Agents is not doing well

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Action of Anne reading the letter is represented as action model

$$(\textbf{Read,p})$$

An example of a valid formula in this model is

$$C_{\alpha,b}(K_\alpha p \vee K_\alpha \neg p)$$

Basic Example Analysis (2/3)

Symbolising the state transition induced by action (**Read,p**) as:

$$0 \text{ —— } \alpha, b \text{ —— } \underline{1} \xrightarrow{\textbf{(Read,p)}} 0 \text{ —— } b \text{ —— } \underline{1}$$

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- Action **p** has precondition atom p, pre(**p**)$= p$
- Action **np** has precondition atom $\neg p$, pre(**np**)$= \neg p$

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- Action **p** has precondition atom p, pre(**p**)$= p$
- Action **np** has precondition atom $\neg p$, pre(**np**)$= \neg p$

So action model (**Read,p**) is symbolised as:

$$\textbf{np} \text{ ——— } b \text{ ——— } \underline{\textbf{p}}$$

Basic Example Analysis (3/3)

The partition of possible actions {**p,np**} for our agents is:

- Anne: {**p**}, {**np**}
- Bill: {**p,np**}

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Anne can distinguish two actions, she can also distinguish the results from those actions, this is called *perfect recall*. Bill cannot.

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$$(0, \mathbf{np}) \sim_b (1, \mathbf{p}) \text{ are the same for Bill}$$

Generalising..

Two states are indistinguishable for an agent if and only if they resulted from two indistinguishable actions executed in two already indistinguishable states.

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$$(s, \mathbf{s}) \sim_\alpha (t, \mathbf{t}) \text{ iff } s \sim_\alpha t \text{ and } \mathbf{s} \sim_\alpha \mathbf{t}$$

We allow $(s, \mathbf{s})$ pairs such that s can be executed in $\mathbf{s}$

$$M, s \models \mathrm{pre}(\mathbf{s})$$

Construction as a Restricted Modal Product

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Modal Product

A modal product of two modal structures is formed by taking the Cartesian product of their domains.

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Modal Product

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Restricted Modal Product

Only allow (s, s) pairs where s can be executed in s.

Basic Example Analysis (as modal product)

In the case of the model $(Letter, 1)$ and the action model (**Read, p**), the result of computing the restricted modal product is:

0 —— α, b —— $\underline{1}$

$\times \quad \Longrightarrow \quad 0$ —— b —— $\underline{1}$

np —— b —— $\underline{\mathbf{p}}$

Basic Example Analysis (as modal product)

In the case of the model $(Letter, 1)$ and the action model (**Read, p**), the result of computing the restricted modal product is:

$0 \text{ —— } \alpha, b \text{ —— } \underline{1}$

$\times \qquad \Longrightarrow \qquad 0 \text{ —— } b \text{ —— } \underline{1}$

$\mathbf{np} \text{ —— } b \text{ —— } \underline{\mathbf{p}}$

Because, $Letter, 0 \models \neg p$ and $Letter, 1 \models p$,

but $(0, \mathbf{np}) \not\sim_\alpha (1, \mathbf{p})$, as $\mathbf{np} \not\sim_\alpha \mathbf{p}$.

Syntactic or Semantic objects?

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Syntactic Object

The preconditions of these 'domain' objects are formulas, so the action model therefore is nothing but some operator with these formulas as arguments, that constructs a more complex formula.

Action Models as Syntactic Objects

- Naming frames, so the structure of the name refers to the structure of the action model.
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  - not the action models themselves are named, but the frames underlying them
- Enumerating frames, inductively
  - A countable supply of elements of the domain (action points)
  - Finite set of agents.
  - Set of pointed frames is enumerable, so the set of names for such frames is also enumerable

Action Models as Semantic Objects

- Replace the preconditions of action points, with semantic propositions
- Semantic proposition $[\![\phi]\!]_M$ stands for $\{s \in \mathcal{D}(M) | M, s \models \phi\}$, where $[\![\phi]\!]$ can be seen as:
  - function from epistemic models to subsets of their domains
  - function from epistemic states to $\{0, 1\}$
- Propositions $[\![\phi]\!]$ are inductively defined
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Exapmle: In action model (**Read,p**), we will write $[\![\text{pre}]\!](\mathbf{p}) = [\![p]\!]$

Contents

Action Model

Action Model

Let $\mathcal{L}$ be any logical language for given parameters agents A and atoms P. An *S5 action model* M is a structure $\langle \mathsf{S}, \sim, \mathsf{pre} \rangle$ such that:

- S is a domain of action points
- for each $\alpha \in A$, $\sim_\alpha$ is an equivalence relation on S
- pre : $\mathsf{S} \to \mathcal{L}$ is a preconditions function that assigns a precondition $\mathsf{pre(s)} \in \mathcal{L}$ to each $\mathsf{s} \in \mathsf{S}$

A *pointed S5 action model* is a structure (M, s) with $\mathsf{s} \in \mathsf{S}$.

Syntax of Action Model Logic

Language of action model logic

Given agents A and atoms P.
The language of action model logic $\mathcal{L}_{KC\otimes}(A, P)$ is the union of the formulas $\varphi \in \mathcal{L}^{stat}_{KC\otimes}(A, P)$ and the actions $\alpha \in \mathcal{L}^{act}_{KC\otimes}(A, P)$ defined by:

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where $p \in P$, $\alpha \in A$, $B \subseteq A$, and (M, s) a pointed action model

- with a finite domain S, and
- s.t. for all $\mathsf{t} \in \mathsf{S}$, the precondition $\mathsf{pre}(\mathsf{t})$ is a $\mathcal{L}^{stat}_{KC\otimes}(A, P)$-formula that has already been constructed.

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Note, $\langle\alpha\rangle\varphi$ is defined as $\neg[\alpha]\neg\varphi$.

Examples (1/2)

Given agents A and atoms P.

Skip

Epistemic action *skip* is defined as

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Public announcement

The action model **pub**(φ), that stands for the truthful public announcement of φ, is defined as

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The action model $\mathbf{pub}(\varphi)$, that stands for the truthful public announcement of φ, is defined as $(\langle\{\text{pub}\}, \sim, \text{pre}\rangle, \text{pub})$, such that $\text{pre}(\text{pub}) = \varphi$, and $\text{pub} \sim_\alpha \text{pub}$ for all $\alpha \in A$.

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Exercise 1

Show that epistemic action (Read, p) from the basic example is a well-formed epistemic action in the language $\mathcal{L}^{act}_{KC\otimes}(\{\alpha, b\}, \{p\})$

Composition

Composition of action models

Let $\mathsf{M} = \langle \mathsf{S}, \sim, \mathsf{pre} \rangle$ and $\mathsf{M}' = \langle \mathsf{S}', \sim', \mathsf{pre}' \rangle$ be two action models in $\mathcal{L}^{act}_{KC\otimes}(A, P)$. Then their composition $(\mathsf{M}; \mathsf{M}')$ is the action model $\langle \mathsf{S}'', \sim'', \mathsf{pre}'' \rangle$ such that:

- $\mathsf{S}'' = \mathsf{S} \times \mathsf{S}'$
- $(\mathsf{s}, \mathsf{s}') \sim''_\alpha (\mathsf{t}, \mathsf{t}')$ iff $\mathsf{s} \sim_\alpha \mathsf{t}$ and $\mathsf{s}' \sim'_\alpha \mathsf{t}'$
- $\mathsf{pre}''((\mathsf{s}, \mathsf{s}')) = \langle \mathsf{M}, \mathsf{s} \rangle \mathsf{pre}'(\mathsf{s}')$

Semantics of Action Model Logic

Semantics of formulas and actions

Given are epistemic state (M, s) with $M = \langle S, \sim, V \rangle$, action model $\mathsf{M} = \langle \mathsf{S}, \sim, \mathsf{pre} \rangle$, and $\varphi \in \mathcal{L}^{stat}_{KC\otimes}(A, P)$ and $\alpha \in \mathcal{L}^{act}_{KC\otimes}(A, P)$.

$$
\begin{array}{lll}
M, s \models p & \text{iff} & s \in V_p \\
M, s \models \neg\varphi & \text{iff} & M, s \not\models \varphi \\
M, s \models \varphi \wedge \psi & \text{iff} & M, s \models \varphi \text{ and } M, s \models \psi \\
M, s \models K_\alpha\varphi & \text{iff} & \text{for all } \mathsf{s}' \in \mathsf{S} : s \sim_\alpha \mathsf{s}' \text{ implies } M, s' \models \varphi \\
M, s \models C_B\varphi & \text{iff} & \text{for all } \mathsf{s}' \in \mathsf{S} : s \sim_B \mathsf{s}' \text{ implies } M, s' \models \varphi \\
M, s \models [\alpha]\varphi & \text{iff} & \text{for all } M', s' : \ (M, s)[\![\alpha]\!](M', s') \\
 & & \quad \text{implies } M', s' \models \varphi \\
(M, s)[\![\mathsf{M}, \mathsf{s}]\!](M', s') & \text{iff} & M, s \models \mathsf{pre}(\mathsf{s}) \text{ and} \\
 & & (M', s') = (M \otimes \mathsf{M}, (s, \mathsf{s})) \\
[\![\alpha \cup \alpha']\!] \ = \ [\![\alpha]\!] \cup [\![\alpha']\!] & &
\end{array}
$$

Semantics of Action Model Logic

Restricted Modal Product

$M' = (M \otimes \mathrm{M})$ is a restricted modal product of an epistemic model and an action model, defined as $M' = \langle S', \sim', V' \rangle$ with:

- $S' = \{(s, \mathrm{s}) \mid s \in S, \mathrm{s} \in \mathrm{S} \text{ and } M, s \models \mathrm{pre}(\mathrm{s})\}$
- $(s, \mathrm{s}) \sim'_\alpha (t, \mathrm{t})$ iff $s \sim_\alpha t$ and $\mathrm{s} \sim_\alpha \mathrm{t}$
- $(s, \mathrm{s}) \in V'_p$ iff $s \in V_p$

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The set of valid formulas from $\mathcal{L}^{stat}_{KC\otimes}$ without common knowledge under the above semantics will be denoted the *action model validities*, or *AM*.

The set of validities from the full language $\mathcal{L}^{stat}_{KC\otimes}$ is *AMC*.

Semantics of Action Model Logic

Note

$M, s \models \langle\alpha\rangle\varphi$ iff there is a M', s' : $(M, s)[\![\alpha]\!](M', s')$ and $M', s' \models \varphi$

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$M, s \models \langle\alpha\rangle\varphi$ iff there is a M', s' : $(M, s)[\![\alpha]\!](M', s')$ and $M', s' \models \varphi$

Specifically:

$M, t \models \langle M, s\rangle \mathrm{pre}'(\mathrm{s}')$ iff $M, t \models \mathrm{pre}(\mathrm{s}) \wedge [\mathrm{M}, \mathrm{s}]\mathrm{pre}'(\mathrm{s}')$

Some Propositions

Proposition 1

Let $(\mathsf{M},\mathsf{s}),(\mathsf{M}',\mathsf{s}') \in \mathcal{L}^{act}_{KC\otimes}(A,P)$, and $\varphi \in \mathcal{L}^{stat}_{KC\otimes}(A,P)$. Then $[(\mathsf{M},\mathsf{s});(\mathsf{M}',\mathsf{s}')]\varphi$ is equivalent to $[\mathsf{M},\mathsf{s}][\mathsf{M}',\mathsf{s}']\varphi$.

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Proposition 2

Let $\alpha, \beta, \gamma \in \mathcal{L}^{act}_{KC\otimes}(A, P)$, then:

- $((\alpha \cup \beta)\ ;\ \gamma)$ equals $((\alpha\ ;\ \gamma) \cup (\beta\ ;\ \gamma))$
- $(\alpha\ ;\ (\beta \cup \gamma))$ equals $((\alpha\ ;\ \beta) \cup (\alpha\ ;\ \gamma))$

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Proposition 3

Let$\alpha,\beta \in \mathcal{L}^{act}_{KC\otimes}(A,P)$. Then $[\alpha \cup \beta]\varphi$ is equivalent to $[\alpha]\varphi \wedge [\beta]\varphi$.

A Corollary and Exercises

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All expressions $[\alpha]\varphi$ are equivalent to some conjunction $\bigwedge[\mathsf{M}, \mathsf{s}]\varphi$.

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- $\varphi \rightarrow [\mathsf{skip}]\varphi$ is valid
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- $[\mathsf{crash}]\perp$ is valid

Exercise 3 (Action model for **mayread**)

Give an action model for the epistemic action **mayread**, where Bill considers it possible that Anne may have read the letter, and where, actually, she doesn't.

Some More Exercises

Exercise 4 (Action model for bothmayread)

Give an action model for the epistemic action **bothmayread**, where both Anne and Bill consider it possible that the other may have read the letter, and where, actually, both read the letter.

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Exercise 5 (Action composition)

Given the epistemic state (Letter, 1) where both Anne and Bill do not know p, and where p is true, first Anne reads the letter ($\text{Read}_{\alpha}, \text{p}\alpha$) and then Bill reads the letter ($\text{Read}_{b}, \text{pb}$).
Compute the composition of Read_{α} and Read_{b}.

Limitations of finite models

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Example: The 'epistemic riddle' concerning consecutive numbers.