Algorithmic Game Theory

Auction theory in practice

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Allocation rules and truthful mechanisms

- We recall first some definitions we saw in previous lectures
- Consider a mechanism with allocation rule **x**

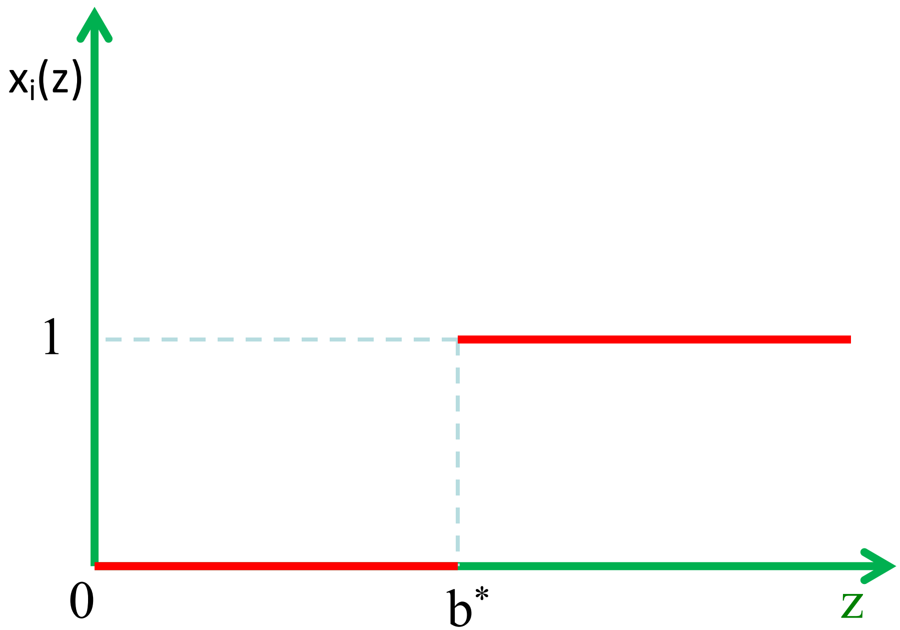
- Definition: An allocation rule is monotone if for every i, and every profile $\mathbf{b}_{-i}$, the allocation $x_i(z, \mathbf{b}_{-i})$ to i is non-decreasing in z
  - i.e., bidding higher can only get you more stuff

[Myerson '81]

- Theorem: For every single-parameter environment,
  - An allocation rule **x** can be turned into a truthful mechanism if and only if it is monotone
  - If **x** is monotone, then there is a unique payment rule **p**, so that (**x**, **p**) is a truthful mechanism

Myerson's lemma and payment formula

- For the payment rule, we need to look for each bidder at the allocation function $x_i(z, \mathbf{b}_{-i})$
- For the single-item truthful auction:
  - Fix $\mathbf{b}_{-i}$ and let $b^* = \max_{j \neq i} b_j$



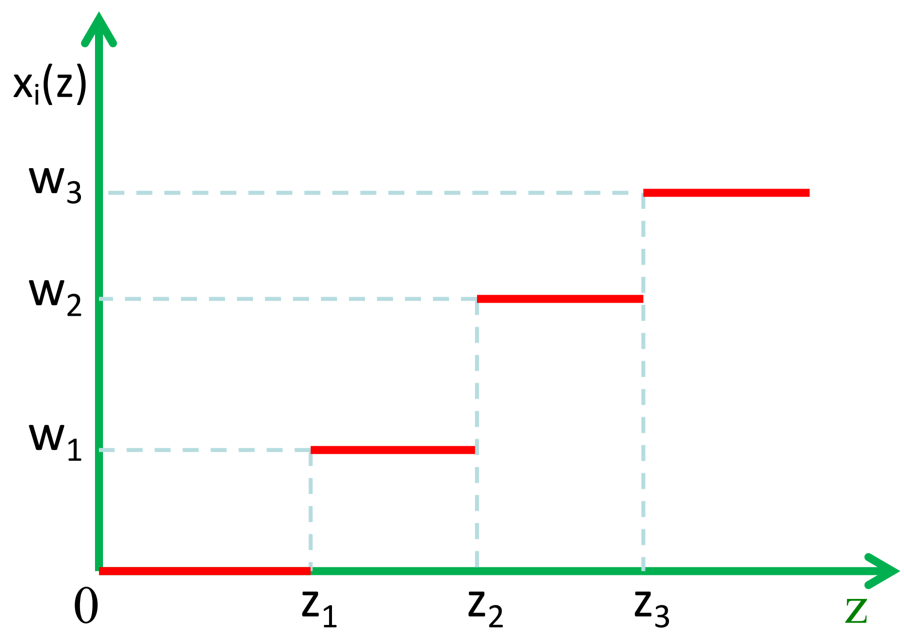
Facts:

- For any fixed $\mathbf{b}_{-i}$, the allocation function is piecewise linear with 1 jump
- The Vickrey payment is precisely the value at which the jump happens
- The jump changes the allocation from 0 to 1 unit

Myerson's lemma and payment formula

For most scenarios of interest

- The allocation is piecewise linear with multiple jumps
- The jump determines how many extra units the bidder wins

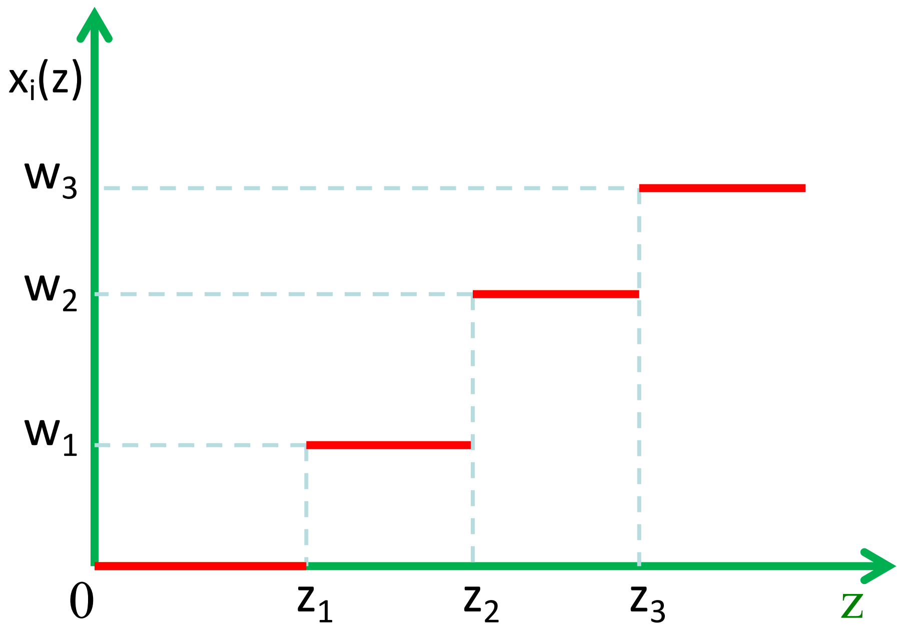


- Suppose bidder i bids b_i
- Look at the jumps of $x_i(z, b_{-i})$ in the interval $[0, b_i]$
- Suppose we have k jumps
- Jump at $z_1 = w_1$
- Jump at $z_2 = w_2 - w_1$
- Jump at $z_3 = w_3 - w_2$
- …
- Jump at $z_k = w_k - w_{k-1}$

Myerson's lemma and payment formula

For most scenarios of interest

- The allocation is piecewise linear with multiple jumps
- The jump determines how many extra units the bidder wins

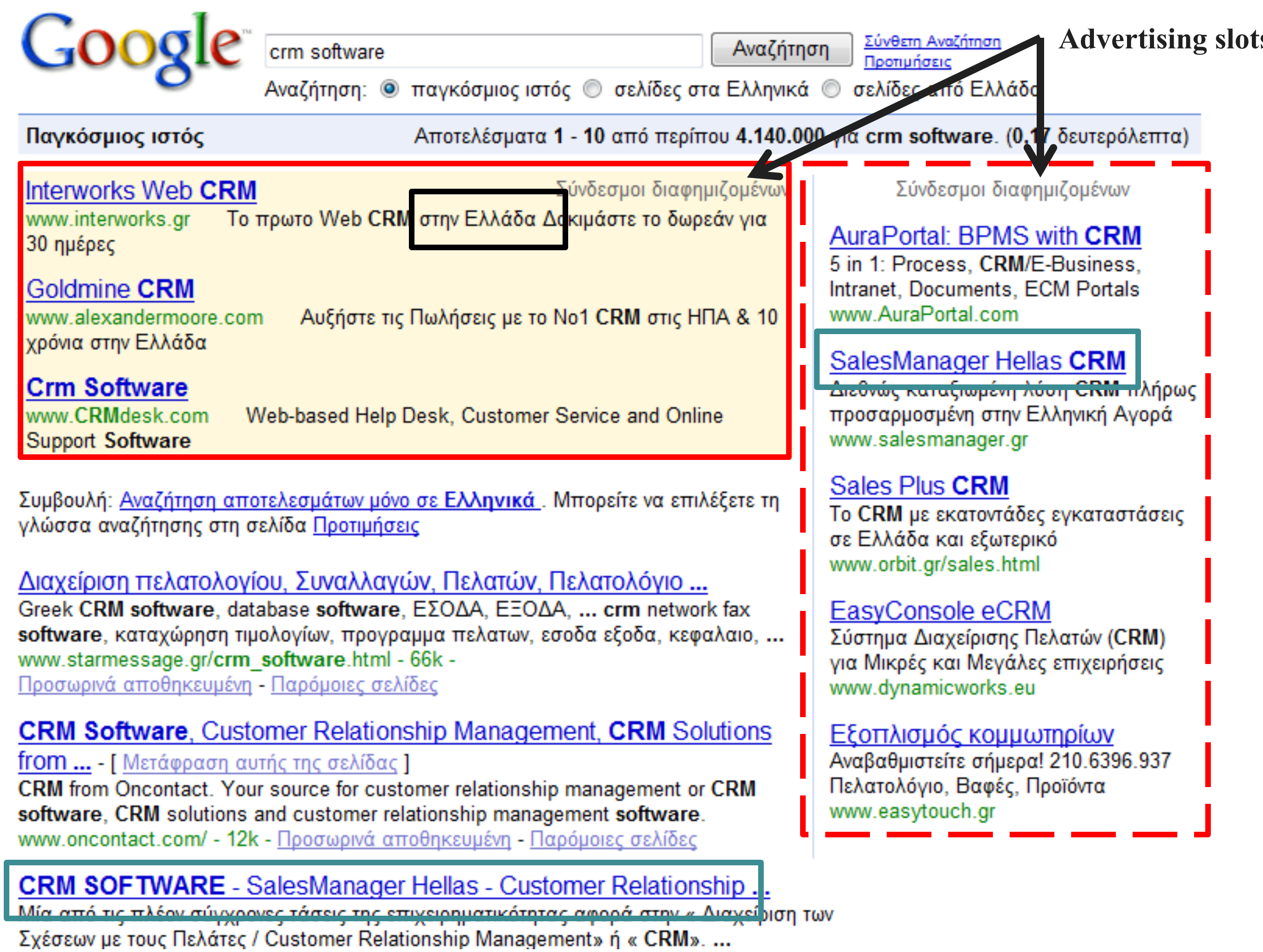


Payment formula

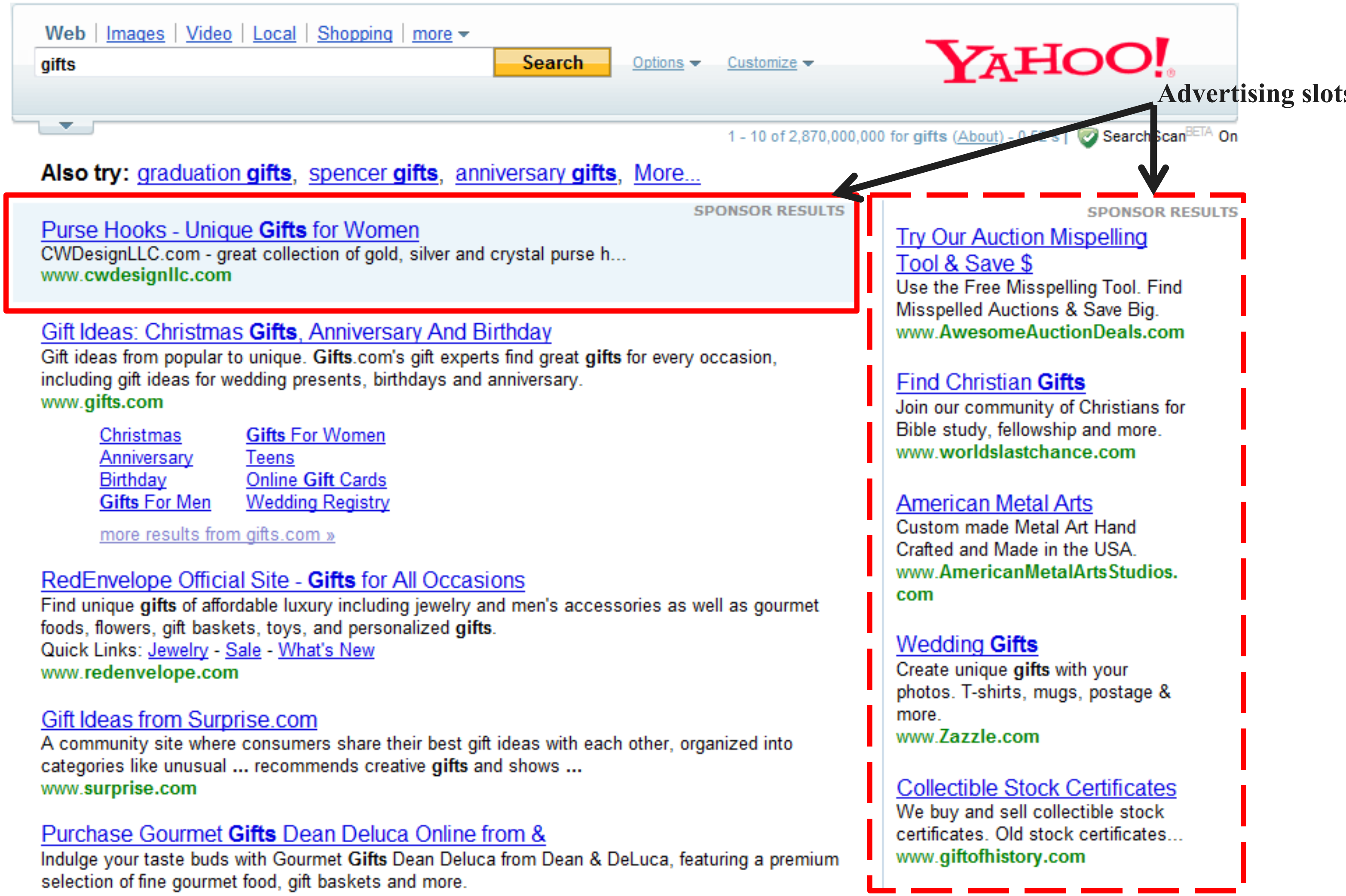
- For each bidder i at a profile b, find all the jump points within $[0, b_i]$
- $p_i(b) = \Sigma_j\, z_j \cdot [\text{jump at } z_j]$
  $= \Sigma_j\, z_j \cdot [w_j - w_{j-1}]$
- The formula can also be generalized for monotone but not piecewise linear functions

Sponsored Search Auctions

What is sponsored search?



What is sponsored search?



How does it work?

- For a fixed search term (e.g. *ipod*)
  - n advertisers
  - k slots (typically k << n)
  - An auction is run for every single search
  - Each advertiser (bidder) is interested in getting himself displayed in one of the slots
    - And usually they prefer a slot as high up as possible
  - Same auction is also run for related keywords (e.g. "buy ipod", "cheap ipod", "ipod purchase", …)
    - The advertiser can determine for which phrases to participate

How does it work?

- Bidders submit an initial budget which they can refresh weekly or monthly
- Bidders also submit an initial bid which they can adjust as often as they wish
- The auction selects the winners to be displayed
- Different charging models exist: Pay Per Click, Pay Per Impression, Pay Per Transaction
- Currently, most popular is Pay Per Click
- A bidder is charged only if someone clicks on the bidder's ad

The Actors

- The Search engine:
  - Wants to make as much revenue as possible
  - At the same time, wants to make sure users receive meaningful ads and bidders do not feel that they were overcharged
  - Big percentage of Google's revenue has been due to these auctions!

- The Bidders:
  - Want to occupy a high slot and pay as little as possible

- The Searchers:
  - Want to find the most relevant ads with respect to what they are looking for

Analyzing sponsored search auctions

- We will focus on the bidders' side
- Model parameters for each bidder i
  - Private information: v_i = maximum amount willing to pay per click = value/happiness derived from a click (private information)
  - Each bidder i submits a bid b_i for willingness to pay per click (b_i may differ from v_i)
  - We will ignore the budget parameter
    - In many cases, it is large enough and cannot affect the game
  - Hence, we have a single-parameter problem

Analyzing sponsored search auctions

- We will focus on the bidders' side
- Model parameters for each slot j
  - α_j = Click-through rate (CTR) of slot j = probability that a user will click on slot j
  - Assume it is independent of who occupies slot j
    - We can generalize to the case where the rates are weighted by a quality score of the advertiser who takes each slot
  - The search engines update regularly the CTRs and statistics show that

$$\alpha_1 \geq \alpha_2 \geq \alpha_3 \geq \ldots \geq \alpha_k$$

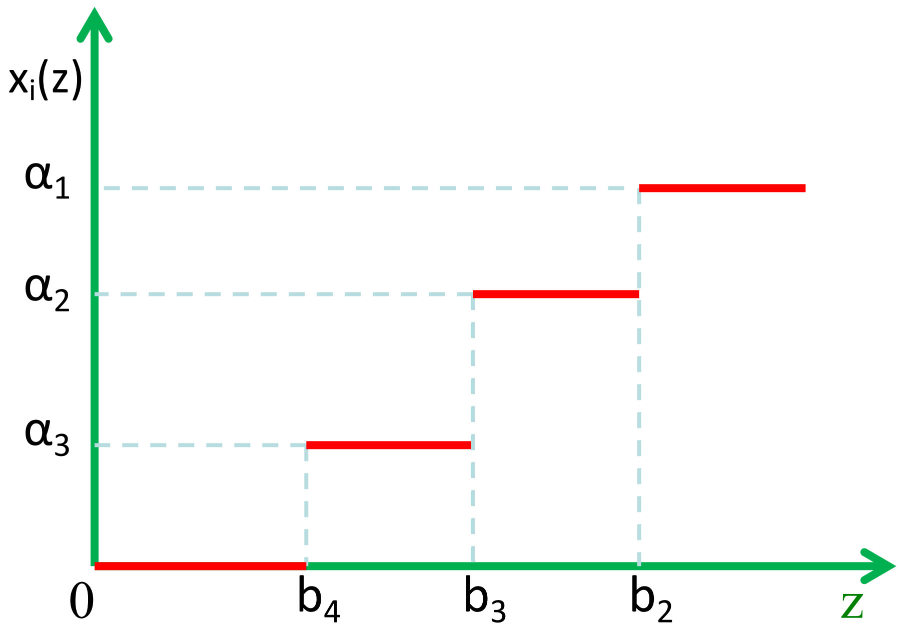
  - Users tend to click on higher slots
    - Validation also by eye-tracking experiments

Analyzing sponsored search auctions

- How shall we allocate the k slots to the n bidders?
- Most natural allocation rule: for i=1 to k, give to the i-th highest bidder the i-th best slot in terms of CTR
  - Remaining n-k bidders do not win anything
- For convenience, assume that $b_1 \geq b_2 \geq b_3 \geq \ldots \geq b_n$
- Expected value of a winning bidder i: $\alpha_i v_i$
- Is this rule monotone?
- Yes, bidding higher can only get you a better slot
- Hence we can apply Myerson's formula to find the payment rule
- For each bidder i, let $x_i(b_i, b_{-i}) \in \{0, \alpha_k, \alpha_{k-1}, \ldots, \alpha_1\}$

Myerson's lemma for sponsored search auctions

- Let's analyze the highest bidder with bid b_1
- Suppose we have 3 slots and n>3 bidders



- Look at the jumps of x_i in the interval $[0, b_1]$
- Jump at $b_4 = \alpha_3$
- Jump at $b_3 = \alpha_2 - \alpha_3$
- Jump at $b_2 = \alpha_1 - \alpha_2$

Total payment:

$b_4\alpha_3 + b_3 (\alpha_2 - \alpha_3) + b_2 (\alpha_1 - \alpha_2)$

Myerson's lemma for sponsored search auctions

- More generally, for the i-th highest bidder, there will be k-i+1 jumps

$$p_i(\mathbf{b}) = \sum_{j=i}^{k} b_{j+1}[\alpha_j - \alpha_{j+1}]$$

- Under pay-per-click, no actual payment takes place at the end of every auction, unless there is a click by a user
- Need to scale so that expected per-click payment is $p_i(\mathbf{b})$
- Proposed per-click payment to bidder in i-th slot: $p_i(\mathbf{b})/\alpha_i$
- By Myerson, no other payment can achieve truthfulness with the same allocation rule

Sponsored search auctions in practice

- In practice most engines do not use the payment of Myerson's lemma
- But they use the same allocation rule
- The Generalized Second Price Mechanism (GSP) - initial version:
  - The search engine ranks the bids in decreasing order: $b_1 \geq b_2 \geq \ldots \geq b_n$
  - The i-th highest bidder takes the i-th best slot
  - Every time there is a click on slot *i*, bidder *i* pays b_{i+1}
  - There is also a reserve price (opening bid), initially the same for every keyword ($0.1), later became keyword-dependent

The Generalized Second Price Mechanism (GSP)

- A better version:
  - The search engine keeps a quality score q_i for each bidder *i*
    - Yahoo, Bing (till a few years ago): q_i is the click-through rate of *i* (probability of a user clicking on an ad of bidder *i*)
    - Google: q_i depends on click-through rate, relevance of text and other factors
  - The search engine ranking is in decreasing order of $q_i \times b_i$
    $q_1 \times b_1 \geq q_2 \times b_2 \geq \ldots \geq q_n \times b_n$
  - The first k bidders of the ranking are displayed in the k slots
  - Every time there is a click on slot *i*, bidder *i* pays the minimum bid required to keep his position, i.e. $(q_{i+1} \times b_{i+1})/q_i$

$$q_i \cdot x_i \geq q_{i+1} b_{i+1} \Rightarrow x_i \geq \frac{q_{i+1} b_{i+1}}{q_i}$$

The Generalized Second Price Mechanism (GSP)

- Myerson's lemma implies GSP cannot be truthful
  - Otherwise, its payment rule would coincide with the Myerson formula
- The deployment of GSP was probably just an educated guess
  - As an attempt to generalize the Vickrey auction and use something simple that looked close to truthful!
- Nevertheless...
  - For a long period, revenue from GSP was 95% of Google's revenue
  - Still nowadays an important percentage of search engines' revenue
- Theoretical analysis of GSP: later in this lecture

Multi-unit auctions

Multi-unit Auctions

Auctions for selling multiple identical units of a single good

In practice:

- US Treasury notes, bonds
- UK electricity auctions (output of generators)
- Spectrum licences
- Various online sales

Multi-unit Auctions

Online sites offering multi-unit auctions

- US
  - www.onlineauction.com
- UK
  - uk.ebid.net
- Greece
  - www.ricardo.gr
  - Actually not any more…
- …

Some Notation

- *n* bidders
- *k* available units of an indivisible good
- Bidder *i* has valuation function $v_i : \{0, 1, \ldots, k\} \rightarrow R$
  - $v_i(j)$ = value of bidder *i* for obtaining *j* units
- Representation with marginal valuations:
  - $m_i(j) = v_i(j) - v_i(j-1)$ = additional value for obtaining the *j*-th unit, if already given *j-1* units
  - $(m_i(1), m_i(2), \ldots, m_i(k))$: vector of marginal values

Some Valuation Classes

- In the multi-unit setting, a valuation v_i is *submodular* iff

  $\forall\ x \le y,\quad v_i(x + 1) - v_i(x) \ge v_i(y + 1) - v_i(y)$

  - Hence: $m_i(1) \ge m_i(2) \ge \ldots \ge m_i(k)$ (decreasing marginal values)
- A valuation v_i is *subadditive* iff

  $\forall\ x, y,\quad v_i(x + y) \le v_i(x) + v_i(y)$
- In many multi-unit auctions, bidders are asked to submit a submodular valuation
  - Makes sense due to the saturation of getting more and more units
- Valuation compression: Even if bidders are not submodular, they would still have to express their preferences by a submodular function

A Bidding Format for Multi-unit Auctions

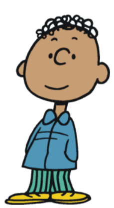
- Used in various multi-unit auctions [Krishna ' 02, Ch. 12-13, Milgrom ' 04, Ch. 7]

1. The auctioneer asks each bidder to submit a vector of decreasing marginal bids
   - $\boldsymbol{b_i} = (b_i(1), b_i(2),\dots, b_i(k))$
   - $b_i(1) \geq b_i(2) \geq \dots \geq b_i(k)$
2. The bids are ranked in decreasing order and the *k* highest win the units

Simplified format in some cases: Uniform bidding, i.e., ask for a bid per unit + number of units demanded

Example

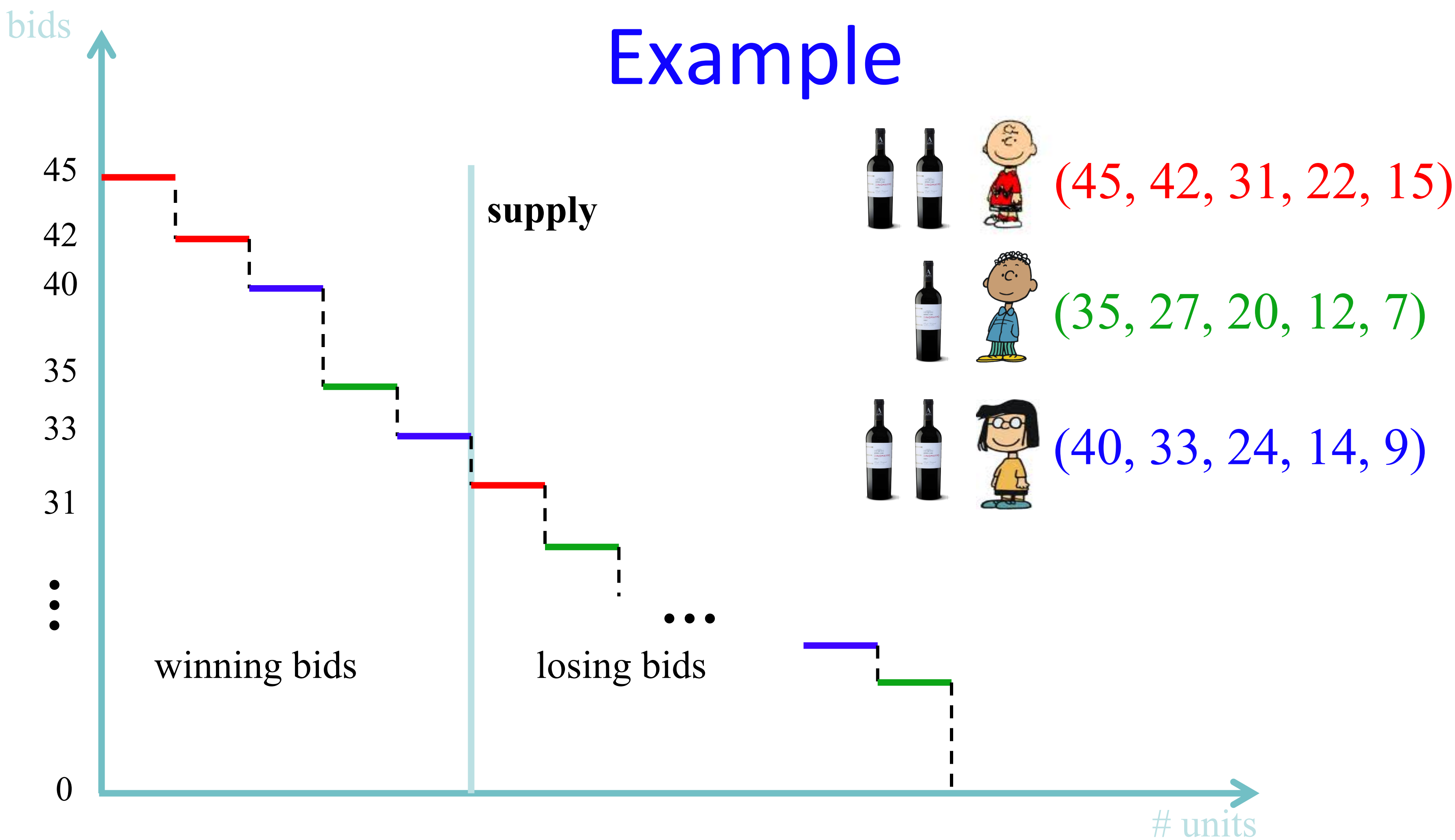
$\mathbf{b_1} = (45, 42, 31, 22, 15)$



$\mathbf{b_2} = (35, 27, 20, 12, 7)$

$\mathbf{b_3} = (40, 33, 24, 14, 9)$

Example



How should we charge the winners?

Pricing Rules

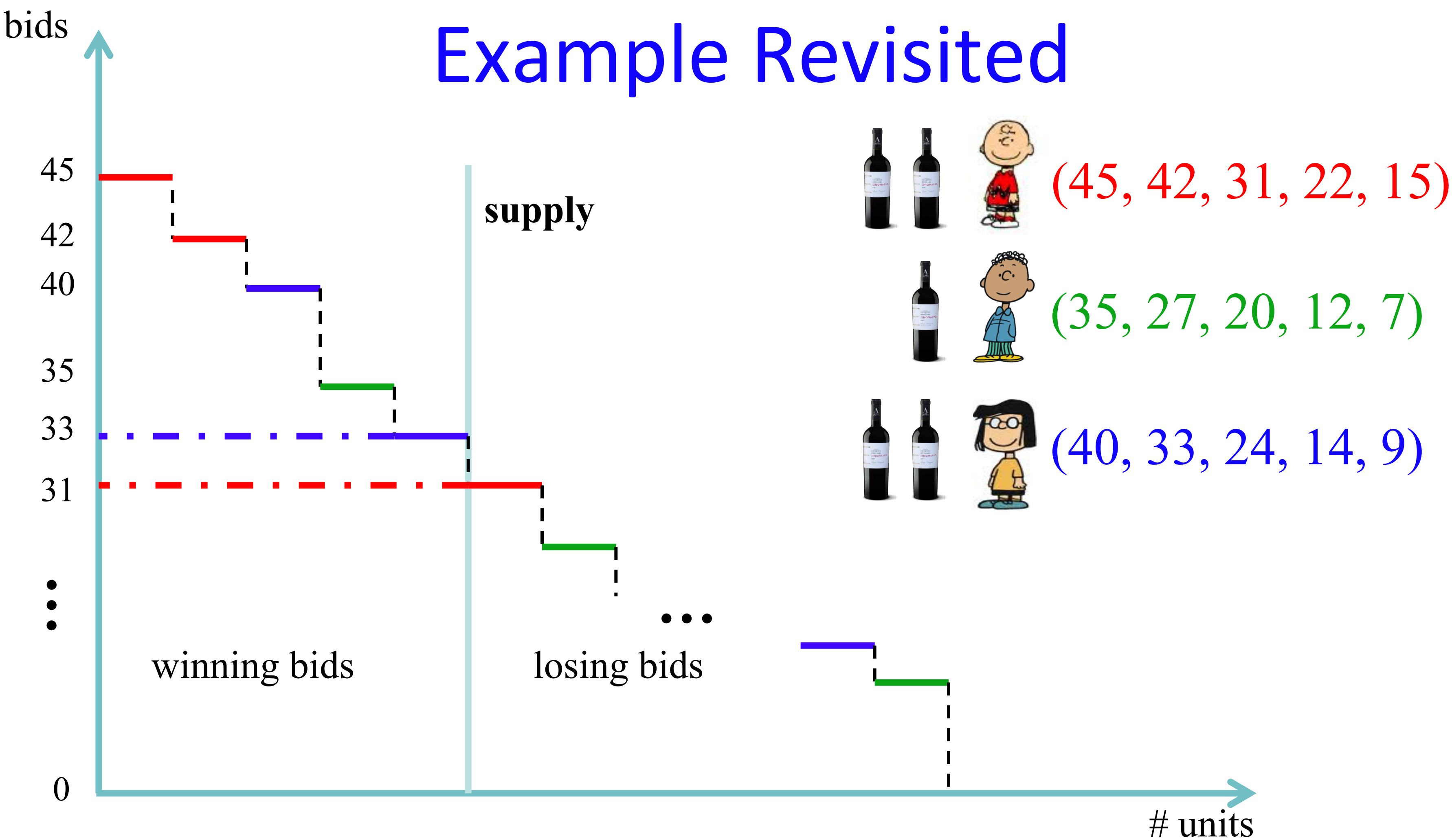
1. Multi-unit Vickrey auction (VCG) [Vickrey ’61]
   - Each bidder pays the externality he causes to the others
   - Generalization of single-item 2nd price auction
   - Good theoretical properties, truthful, but barely used in practice
2. Discriminatory Price Auction (DPA)
   - Bidders pay their bids for the units won
   - Generalization of 1st price auction
   - Not truthful, but widely used in practice

Pricing Rules (cont’d)

3. Uniform Price Auction (UPA) [Friedman 1960]
   - Same price for every unit
   - Interval of prices to pick from:

     [highest losing bid, lowest winning bid]
   - This lecture: price = highest losing bid
   - For 1 unit, same as Vickrey auction
   - For ≥ 2 units, not truthful, but widely used in practice (following the campaign of Miller and Friedman in the 90’s)

Example Revisited



Interval of candidate prices for UPA = [31, 33]

Uniform price = 31

Uniform Price vs Discriminatory?

- Debate still going on for treasury auctions
- DPA is thought to raise more revenue (no formal justification though)
- UPA eliminates complaints arising from price discrimination (identical goods should cost the same!)

Equilibrium analysis of non-truthful mechanisms

Non-truthful mechanisms

- As already seen, there are plenty of settings where the mechanism employed is not truthful
  - Sponsored search
  - Auctions for government bonds
  - Some types of auctions for telecom/spectrum licences (e.g., core-selecting auctions)
- Why?
  - Low revenue often achieved by truthful auctions, e.g., by VCG
  - Complexity: Social welfare maximization may turn out too difficult to solve (which is a required step in VCG-based mechanisms)

- [Ausubel, Milgrom '06]: The lovely but lonely Vickrey auction
  - Chapter 1 in the book "Combinatorial Auctions"

Non-truthful mechanisms

- How do we evaluate non-truthful mechanisms?
  - If the bidders are non-truthful, can we argue about the social welfare generated?
- We can think of the equilibria as the most likely outcomes to occur
  - If these games are played frequently, players may end up at an equilibrium by adjusting gradually their strategies
  - Thus, we can take the social welfare or revenue achieved at an equilibrium as an evaluation metric

PoA in auctions

- Consider an auction where v_i = actual valuation function of bidder i
  - It can be either single or multi-parameter
- Let **b** be a pure Nash equilibrium with resulting allocation: $(x_1,\dots, x_n) = (x_1(\mathbf{b}),\dots, x_n(\mathbf{b}))$
- Social Welfare at **b**: $SW(\mathbf{b}) = \Sigma\, v_i(x_i)$
- OPT = Optimal welfare (as determined by the valuations)

$$PoA = \sup_{\mathbf{b}}\ OPT/SW(\mathbf{b})$$

Where the supremum can be either over all pure or over all mixed equilibria

PoA in sponsored search auctions

- PoA can become unbounded in worst case
- [Lahaie '06]: PoA $\leq (\min_{1\leq i\leq k-1} \min\{\alpha_{i+1}/\alpha_i, 1 - (\alpha_{i+2}/\alpha_{i+1})\})^{-1}$
  - For pure equilibria, when we have k≥2 slots
  - Where recall α_i is the CTR of slot i, and assume $\alpha_{k+1} = 0$
- For arbitrary auctions, the ratios of the CTRs can become arbitrarily high
- In some cases, the click data fit well with an exponential decay model (geometric CTRs): $\alpha_i \propto 1/\delta^i$ for a constant δ
  - [Feng, Bhargava, Pennock '07]: δ = 1.428 using various empirical datasets
  - In these cases, PoA $\leq (\min\{1/\delta, 1-1/\delta\})^{-1}$
  - Hence, low inefficiency under geometric CTRs

PoA in sponsored search auctions

- One can also study PoA under restrictions on the set of equilibria under consideration
- E.g., some “bad” equilibria arise when some players overbid and at the same time some high-valued players underbid
- The no-overbidding assumption: Focus on equilibria where $b_i \leq v_i$
  - Such bidders are also referred to as conservative bidders
  - Initiated in [Christodoulou, Kovacs, Schapira ’08], and assumed in several follow up works
- Can PoA be better under no-overbidding?

PoA in sponsored search auctions

- [Paes Leme, Tardos ’10]: Under no-overbidding
  - PoA ≤ 1.618 (= 1 + ϕ) for pure equilibria
  - PoA ≤ 4 for mixed equilibria
- [Lucier, Paes Leme ’11, Caragiannis et al. ’11, ’15]: Currently best known:
  - PoA ≤ 1.28 for pure equilibria
  - PoA ≤ 2.31 for mixed equilibria
- For lower bounds, it is known that PoA ≥ 1.259
- Main conclusion: For conservative bidders, selfish behavior does not lead to socially bad outcomes

Revenue in sponsored search auctions

- Could we have analogous guarantees for revenue instead of social welfare?
  - Harder problem…
- But, some comparisons can be drawn between the use of GSP and VCG
- [Varian '05, Edelman, Ostrovsky, Schwarz '07]: Focus on the class of "locally envy-free equilibria"
  - As a plausible class of equilibria that may arise
  - Analyzed for the simple version of GSP, without the personalized quality score q_i
  - But their results can be stated for the more general setting as well

Revenue in sponsored search auctions

- For convenience, rename the bidders so that the bidder occupying slot j has value v_j and pays price p_j
  - i.e., p_j = bid of bidder in slot j+1
- Definition: The profile **b** = $(b_1, b_2, \ldots, b_n)$ is a locally envy-free equilibrium, if for a bidder at slot s, we have

$$\alpha_s (v_s - p_s) \geq \alpha_j (v_s - p_j) \text{ for every other slot } j$$

- This means no bidder is willing to swap her slot and price with those of another bidder
- In fact, it suffices to check only the neighboring slots
  - Look only at slot s-1 and s+1 for the bidder at slot s
  - Thus the name "locally envy-free"

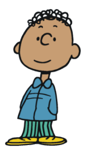
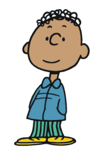
Revenue in sponsored search auctions

- Main theorem in [Varian ’05, Edelman, Ostrovsky, Schwarz ’07]:
  - (i) There exists a no-overbidding locally envy-free equilibrium where allocation + payments coincide with the VCG outcome
  - (ii) The revenue at any locally envy-free equilibrium ≥ VCG revenue (at truthful profile)
- Can be seen as a justification of why GSP is a better choice than VCG for sponsored search auctions
- Although GSP was probably employed by accident, it was a rather good choice!

PoA in multi-unit auctions

- A PoA analysis can be carried out for any other non-truthful auction
- For multi-unit auctions, PoA can be affected by the phenomenon of “demand reduction”
  - [Ausubel, Cramton ’96]: Bidders may have incentives to hide their demand for items in order to achieve a better price

Example of Demand Reduction in UPA

Real profile	Equilibrium profile
 (1, 1, 1)	 (1, 0, 0)
 (2/3, 0, 0)	 (2/3, 0, 0)
 (1/2, 0, 0)	 (1/2, 0, 0)

OPT = 3, SW(**b**) = 13/6 ⇒ PoA ≥ 18/13 for UPA

- Revealing the true profile for bidder 1 results in a relatively high price
- Demand reduction discussed further in [Ausubel, Cramton ’96]

PoA for pure equilibria

Can demand reduction create a huge loss of efficiency?

Theorem:

For the Discriminatory Price Auction (DPA), and **arbitrary** monotone valuations for the bidders, PoA = 1

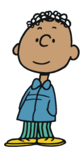
- No need to assume no-overbidding
- All pure Nash equilibria (when they exist) are efficient
- Generalizes what holds for the single-item 1st price auction (recall your first homework!)
- Existence of pure equilibria guaranteed under appropriate tie-breaking rules

PoA for pure equilibria

- The same is not true for UPA
- Example: Consider k units and the profiles:

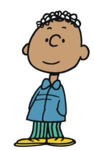
Real profile


(k, 0, 0,…,0)


(1, 1, 1,…, 1)

Equilibrium profile **b**


(1, 1, 1,…, 1)


(0, 0, 0,…, 0)

- OPT = 2k-1
- SW(**b**) = k
- PoA ≥ (2k-1)/k = 2 – 1/k for UPA
- Can it get worse?

PoA for pure equilibria

- For non-conservative bidders, it can get unbounded
- The no-overbidding assumption in UPA:

$$\sum_{j=1}^{s} b_i(j) \leq v_i(s) \; \forall i, \forall s \leq k$$

[Birmpas, Markakis, Telelis, Tsikiridis '17]:

For the Uniform Price Auction (UPA), and for

- Submodular bidders
- No-overbidding pure equilibria,

$$\text{PoA} \leq 2.18$$

- Tight example even for 2 bidders

PoA for mixed equilibria

[de Keijzer, Markakis, Schaefer, Telelis ’13]:
For submodular valuations, the PoA for mixed equilibria is

- $\leq e/e\text{-}1$ for DPA
- $\leq 3.146 < 2e/e\text{-}1$ for UPA

Remarks:
- $3.146\ldots = |W_{-1}(-1/e^2)|$ (Lambert W function)
- Bounds hold both for standard bidding and for the simplified uniform bidding format
- The same bounds also hold for Bayesian games (PoA for Bayes-Nash equilibria)

PoA for mixed equilibria

- Currently known lower bounds: ≈1.1 for DPA, 2.18 for UPA
  - Far from tight in the case of mixed equilibria
- Our proof can be cast into the smoothness framework of [Syrgkanis, Tardos ’13]

$$\Downarrow$$

- Upper bounds carry over to simultaneous and sequential compositions of multi-unit auctions (e.g. combinatorial multi-unit auctions)
- Similar approaches and techniques used in other types of auctions as well (e.g. item-bidding auctions) [Christodoulou, Kovacs, Schapira ’08, Bhawalkar, Roughgarden ’11, Feldman, Fu, Gravin, Lucier ’13]

48

Beyond Submodular Valuations

- [Milgrom '04]: Very little known (i.e., nothing) for non-submodular bidders

- Subadditive valuations: Valuation compression is needed for such bidders

Lemma: Subadditive valuations can be approximated by submodular functions, losing a factor of 2

Subadditive Valuations

Theorem: For subadditive valuations, mixed PoA is at most:

Auction \Bidding	Standard bidding	Uniform bidding
DPA	2	2e/e-1
UPA	4	6.292 < 4e/e-1

- Uniform bidding: same technique as before, using the 2-approximation
- Standard bidding: Adaptation of [Feldman, Fu, Gravin, Lucier '13] into multi-unit auctions
  - Deviation constructed by sampling from the distribution of $\mathbf{b}_{-i}$

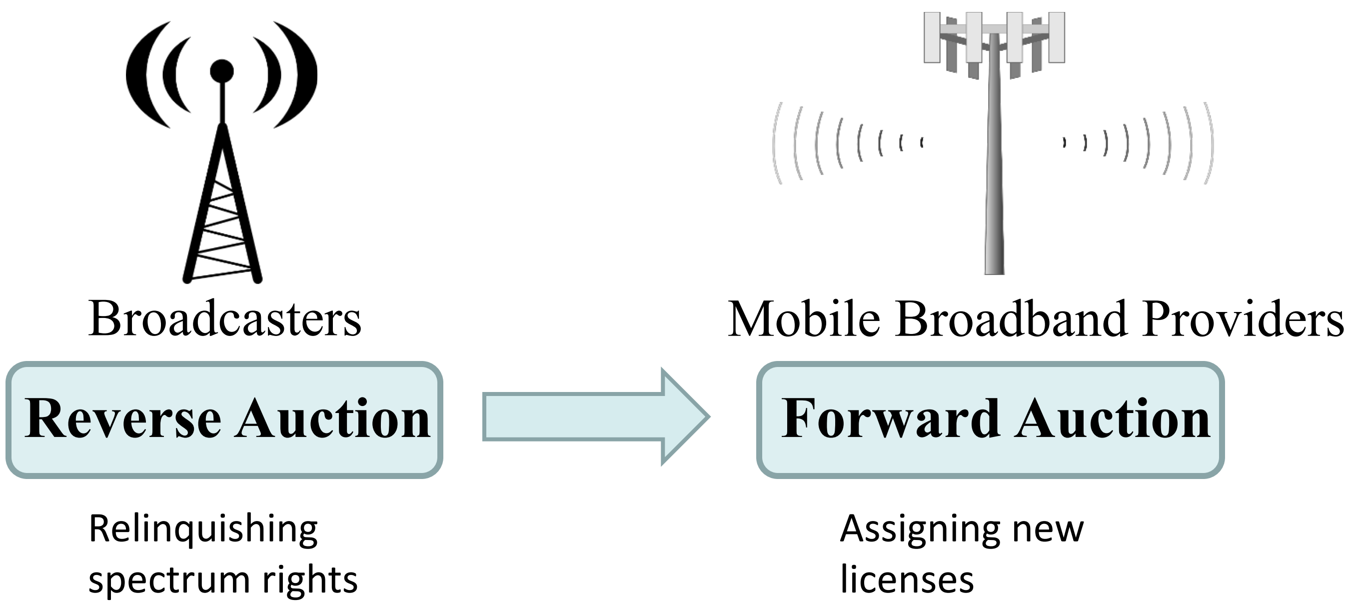
Conclusions on PoA

- Take-home story: simple auction formats used in practice perform quite well w.r.t. social welfare
- Upper bounds:
  - For pure equilibria, almost tight for sponsored search, completely tight for multi-unit auctions
  - Open if we can improve the bounds for mixed equilibria
  - PoA can also become even better if we focus on Nash equilibria in undominated strategies
- Lower bounds:
  - Much harder to get

Examples of truthful auctions in practice

Spectrum Auctions

- Deferred Acceptance Auctions initiated by [Milgrom, Segal '14]
- Motivated by the design of the FCC **"Broadcast Incentive Auction"**



- Commenced on March 2016, closed on April 2017 for repurposing spectrum to align with consumer demand for broadband services

Basic Mechanism Design Setting

Main features:

- A **provider** of some service or resources
- A set of **single-parameter buyers** N = {1, 2, ..., n} interested in (some of) the resources
- Each buyer has a **valuation** $\mathbf{v_i}$
- For each buyer: need to make an **accept/reject** decision
- **Feasible solutions:** Only specific subsets of buyers may be served simultaneously, due to problem constraints (e.g. interference constraints in spectrum auctions)

The framework of Deferred-Acceptance Auctions

- Backward greedy allocation algorithms
- They work in rounds, finalizing the decision for a single bidder in each round
- A_t = set of active bidders at round t
- Score of bidder i at round t: $\sigma_i^{A_t}(b_i, b_{N \setminus A_t})$
  - non-decreasing in b_i
  - Possible dependence on the set A_t (but not on the bids of active bidders)

1. Initially all bidders are **active** (A_1=N)
2. While accepting all active bidders in A_t is **infeasible**
   - **Reject** the bidder *i* with the lowest score
   - $A_{t+1} = A_t \setminus \{i\}$
3. Remaining bidders are **accepted** and pay **threshold prices**

Properties of Deferred-Acceptance Auctions

Incentive guarantees:

- Not hard to show that DA auctions are truthful
- In fact we can have much stronger incentive guarantees

Definition: A mechanism is weakly group-strategyproof if: for any coalition $S \subseteq N$, and any profile b_{-S}, there is no deviation by S, such that all members are strictly better off, i.e., such that:

$$u_i(b_S, b_{-S}) > u_i(v_S, b_{-S}), \text{ for every } i \in S$$

Lemma: DA auctions are weakly group-strategyproof

Properties of Deferred-Acceptance Auctions

Further advantages of DA auctions:

1. Practical and simple to implement **as long as**
   - Scoring function is simple
   - Checking feasibility of a solution is easy
2. They admit an implementation as an ascending clock auction
3. Using the ascending auction implementation:
   - Very easy to argue that truth-telling is a dominant strategy (obvious strategyproofness [Li ’15])
   - Privacy preservation: winners do not reveal their true value

Possible limitations:

1. They do not always guarantee a good approximation to the social welfare
2. Same for other objectives (e.g. revenue)
3. Solution returned may not be a maximal set w.r.t. problem constraints (drawback of backward greedy algorithms)

An illustration

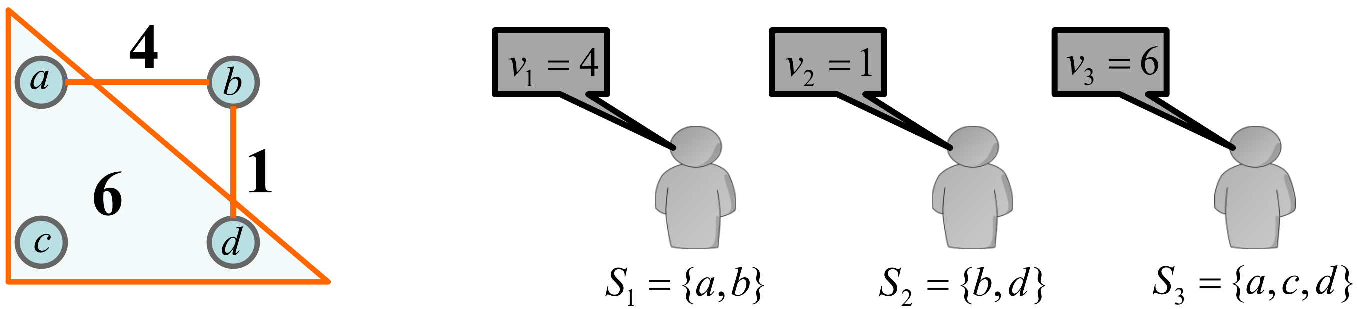
Recall single-minded bidders from previous lectures

- The auctioneer has a set M of items for sale
- Each bidder i is interested in acquiring a specific subset of items, $S_i \subseteq M$ (known to the mechanism)
    - If the bidder does not obtain S_i (or a superset of it), his value is 0
- Each bidder submits a bid b_i for his value if he obtains the set

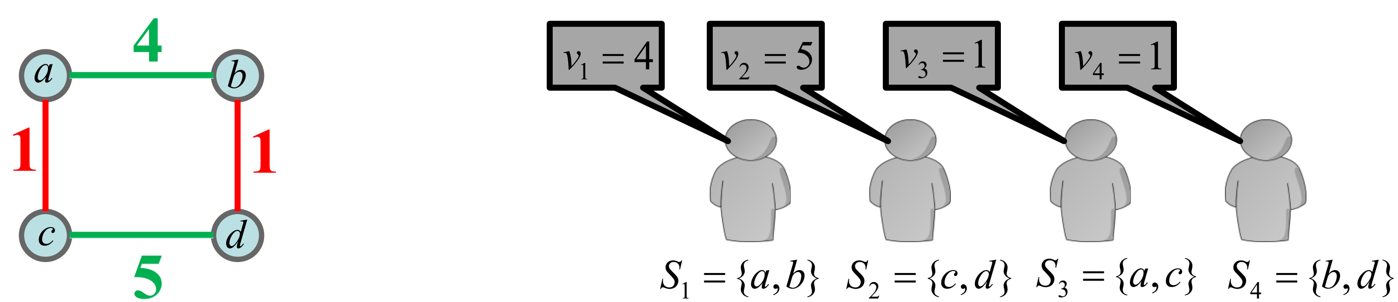
- Motivated by certain spectrum auctions
- Feasible allocations: the auctioneer needs to select winners who do not have overlapping sets

Single-minded bidders

Examples



- In the examle above, the auctioneer can accept only 1 bidder as a winner
- In the example below, the auctioneer can accept up to 2 bidders as winners



A forward greedy algorithm for single-minded bidders

[Lehmann, O' Callaghan, Shoham '01]:

$s_i=|S_i|$

- Order the bidders in decreasing order of $b_i/\text{sqrt}(s_i)$
- Accept each bidder in this order unless overlapping with previously accepted bidders

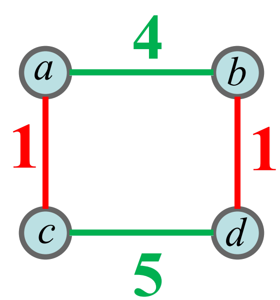
This algorithm achieves

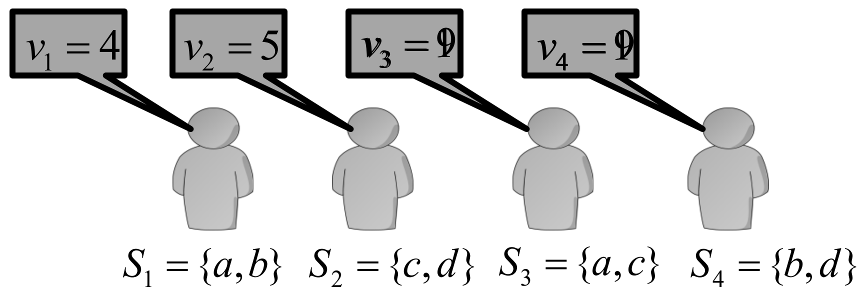
- Monotonicity of the allocation (hence can be made truthful)
- 1/sqrt(m)-approximation, where m = |M|
- 1/d-approximation, where $d = \max_i s_i$

Final conclusion: truthful polynomial time mechanism with the best possible approximation to the social welfare

Coalitions under the forward greedy mechanism

- The forward greedy mechanism is truthful but suppose players could also collude:



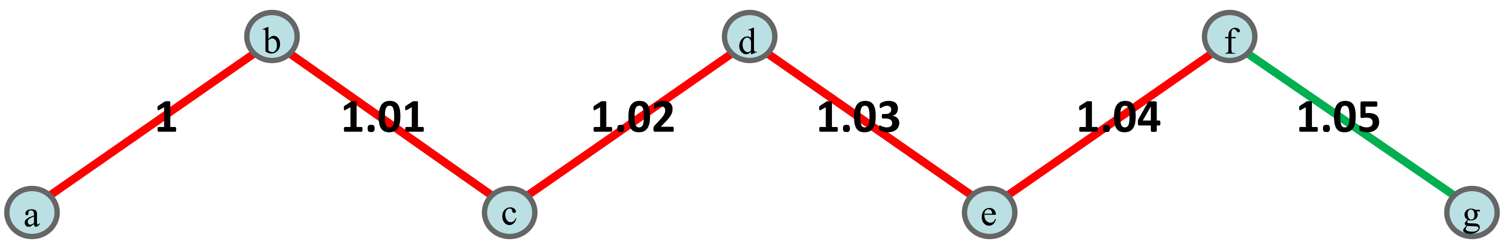


- What would forward greedy do?
    1. Accept bid {c,d}
    2. Reject bids {a,c} and {b,d}
    3. Accept bid {a,b}
    4. Threshold price = 0

- The coalition {3, 4} can change the outcome
- Threshold price still 0
- Both members better off!
- Forward greedy is not group-strategyproof

Scoring Functions for DA auctions

- Can we achieve similar welfare guarantees with backward greedy algorithms?
- How about a DA auction with scoring $\sigma_i(v_i, s_i) = v_i / \sqrt{s_i}$?



- Backward greedy can do much worse than forward greedy
- Use conflict number $\sigma_i(v_i, c_{i,t}) = v_i / c_{i,t}$?
  - $c_{i,t}$ = number of conflicts with other bidders at stage t

[Dutting, Gkatzelis, Roughgarden '14]:

- This does not work either
- Having s_i or $c_{i,t}$ in the denominator, raised to any power cannot achieve an $O(1/d)$ or $\tilde{O}(1/\sqrt{m})$ approximation

Positive results for DA auctions

[Dutting, Gkatzelis, Roughgarden ’14]:

Theorem 1: There exists a DA auction that achieves an approximation ratio of **O(*d*)**

Theorem 2: There exists a DA auction that achieves an approximation ratio of **$O(\sqrt{m}\log m)$**

Main message:

We can have comparable approximations as in forward greedy, but with stronger incentive guarantees!

- And with a more complicated scoring function

Final conclusions

- A wide range of applications
- The full spectrum of incentive guarantees can be seen in practice
  - Non-truthful and bad equilibria (uniform price auction or sponsored search with overbidding)
  - Non-truthful and efficient equilibria (single-item first price auction)
  - Non-truthful and relatively efficient equilibria (sponsored search, uniform price auction, under no-overbidding)
  - Truthful (single-item Vickrey)
  - Weakly group-strategyproof (DA auctions)
- The choice of mechanism deployed may depend on:
  - Traditions and practices used in a specific application domain (not always easy to switch to a new format)
  - Complexity considerations (simplicity is often a must)
  - Legal issues (there exist governmental auctions where social welfare w.r.t. reported bids needs to be maximized)