Helios Extensions For Privacy - Decentralized voting

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NTUA -Advanced topics in cryptography (2020-2021)

Privacy

Modelling Secrecy i

- Secrecy in voting defers from secrecy in messaging
- Ballot privacy is not absolute
- The result leaks information
  - In a unanimous vote, everyone knows how everyone voted
  - In an all-but-one vote, the one that differs knows how everyone else voted
  - The result also yields a probability of a particular vote
  - Important for small voting populations

Modelling Secrecy ii

3 approaches to defining ballot privacy

- Simulation-Based: Securely realize some ideal functionality that is secret
- Entropy-Based: Measure the amount of information the voting scheme leaks
- Game-Based: Model the privacy property as a game between the system and an attacker
  - Easier to work
  - Many attempts that differ on their relation to the result function and other properties of the voting scheme (e.g. verifiability)

Game-based definitions for ballot secrecy

Threat model $\mathcal{A}$ corrupts voters. TA is honest!

Intuition: Indistinguishability games for cryptographic secrecy

Instead of distinguishing between message encryptions the $\mathcal{A}$ tries to distinguish between different voting scenarios

Workflow:

- $\mathcal{C}$ setups the protocol and begins with empty BB
- $\mathcal{A}$ corrupts voters and casts ballots on their behalf
- For an honest voter $\mathcal{A}$ selects 2 choices and hands them to $\mathcal{C}$
- $\mathcal{C}$ flips a coin and selects one of the choices
- $\mathcal{C}$ creates and casts the respective ballot
- $\mathcal{A}$ casts more corrupted ballots
- The $\mathcal{A}$ must guess the coin

This must be combined with verifiability and apply to all possible result functions

Impact of voting rules

Assume the majority result function and candidates $\{a, b\}$

The candidate with majority is declared the winner and a in case of a tie

There are two honest voters and one corrupt voter

In scenario 1: $\{(i_0, a)(i_1, a)\}$ winner: a

In scenario 2: $\{(i_0, a)(i_1, b)\}$ winner: a

$\mathcal{A}$ can distinguish the scenario by casting a a ballot for b

Lack of ballot independence can break ballot privacy i

Ballot independence
A voter cannot repost an exact replica of another voter's ballot in the BB

Assume an election with 3 voters, where V_1, V_2 are honest and V_3 is corrupt.

$$b_1 = (1, (R_1, S_1), \pi_{\mathsf{Vote},1})$$
$$b_2 = (2, (R_2, S_2), \pi_{\mathsf{Vote},2})$$

The voter replays an exact replica of b_1

$$b_3 = (3, (R_1, S_1), \pi_{\mathsf{Vote},1})$$

The candidate that receives more than 2 votes is the one preferred by V_1

Lack of ballot independence can break ballot privacy ii

Alternative implementation: Use malleability and construct a different ballot for the same vote

Countermeasures:

- Ballot weeding
- Strong Fiat-Shamir Heuristic to counter encryption malleability
- Add voter id to the hash function

Despite being naive such attacks provide a sanity check for voting systems

The BPRIV framework

Goal
A private protocol does not leak any more information than what is leaked from the tally.

Main idea
$\mathcal{A}$: Tries to distinguish between two worlds by having access to:

- The 'real' BB that contains honest and adversarial ballots. The adversary sees the real result.
- A 'fake' BB. The adversary sees the real results. Proofs of correctness are simulated.

Overview of BPRIV

$\mathcal{A}$ has access to the following oracles:

- **Board**(b): Retrieve the contents of the BB_b (**Publish**)
- **Vote**(v_0, v_1): Select two votes and post the corresponding ballots to BB_0, BB_1 respectively - ballots are honestly created
- **Cast**(b): Cast a ballot to both BB - ballot is adversarially created
- **Tally**(b): Obtain the result of BB_0. Yield real or simulated proofs

$\mathcal{A}$ can call the oracles **Board**, **Vote**, **Cast** at will

$\mathcal{A}$ can call **Tally** only once

$\mathcal{A}$ must guess which BB is presented

The BPRIV definition i

Algorithm 1: $\mathsf{BPRIV}^{b}_{\mathcal{A},\mathsf{VS}}$

Input : security parameter λ

Output: $\{0, 1\}$

$(\mathsf{pk}_{\mathsf{TA}}, \mathsf{sk}_{\mathsf{TA}}) \leftarrow \mathsf{VS}.\mathbf{Setup}(1^{\lambda})$

$\mathsf{CS} \leftarrow \mathcal{A}()$

$b' \leftarrow \mathcal{A}^{\mathbf{Vote},\mathbf{Cast},\mathbf{Tally},\mathbf{Board}}(\mathsf{pk}_{\mathsf{TA}})$

return $b = b'$

BPRIV

A voting system VS supports ballot privacy if there exists a simulator for $\mathcal{S}$ such that $\forall \mathsf{PPT}\mathcal{A}$:

$$| \Pr\left[\mathsf{BPRIV}^{0}_{\mathcal{A},\mathsf{VS}}(\lambda) = 1\right] - \Pr\left[\mathsf{BPRIV}^{1}_{\mathcal{A},\mathsf{VS}}(\lambda) = 1\right] | = \mathsf{negl}(\lambda)$$

The BPRIV definition ii

Algorithm 2: Oracles for BPRIV definition

Oracle *Vote*(i, vt_0, vt_1)
- $b_0 := \text{Vote}(i, vt_0)$
- $b_1 := \text{Vote}(i, vt_1)$
- if $\text{Valid}(b_0, \text{BB}_0)$ AND $\text{Valid}(b_1, \text{BB}_1)$ then
  - $\text{BB}_0 \Leftarrow b_0$
  - $\text{BB}_1 \Leftarrow b_1$
- else
  - return $\perp$

Oracle *Cast*(i, b, b)
- if $\text{Valid}(b, \text{BB}_{\text{b}})$ then
  - $\text{BB}_0 \Leftarrow b$
  - $\text{BB}_1 \Leftarrow b$
- else
  - return $\perp$

Oracle *Tally*(b)
- if $\text{b} = 0$ then
  - $(T, \pi_T) := \text{Tally}(\text{sk}_{\text{TA}}, \text{CS}, \text{BB}_0)$
  - return (T, π_T)
- else
  - $(T, \pi_T) := \text{Tally}(\text{sk}_{\text{TA}}, \text{CS}, \text{BB}_0)$
  - $\pi'_T := \mathcal{S}(\text{sk}_{\text{TA}}, \text{CS}, \text{BB}_0, \text{BB}_1, T)$
  - return (T, π'_T)

Oracle *Board*(b)
- return $\text{Publish}(\text{BB}_{\text{b}})$

Helios is BPRIV i

Observation
The visible BB consists of tuples $(i, b, \cdot)$ posted using **Cast**, **Vote**

If the tuple originates from **Vote**, the challenger can internally map it to a tuple $(i, b_0, vt_0, b_1, vt_1)$

If the tuple originates from **Cast**, then it can be mapped to $(i, b, \perp, b, \perp)$

Proof overview
A sequence of games beginning from BPRIV^0 and ending to BPRIV^1 with indistinguishable differences for $\mathcal{A}$

Helios is BPRIV ii

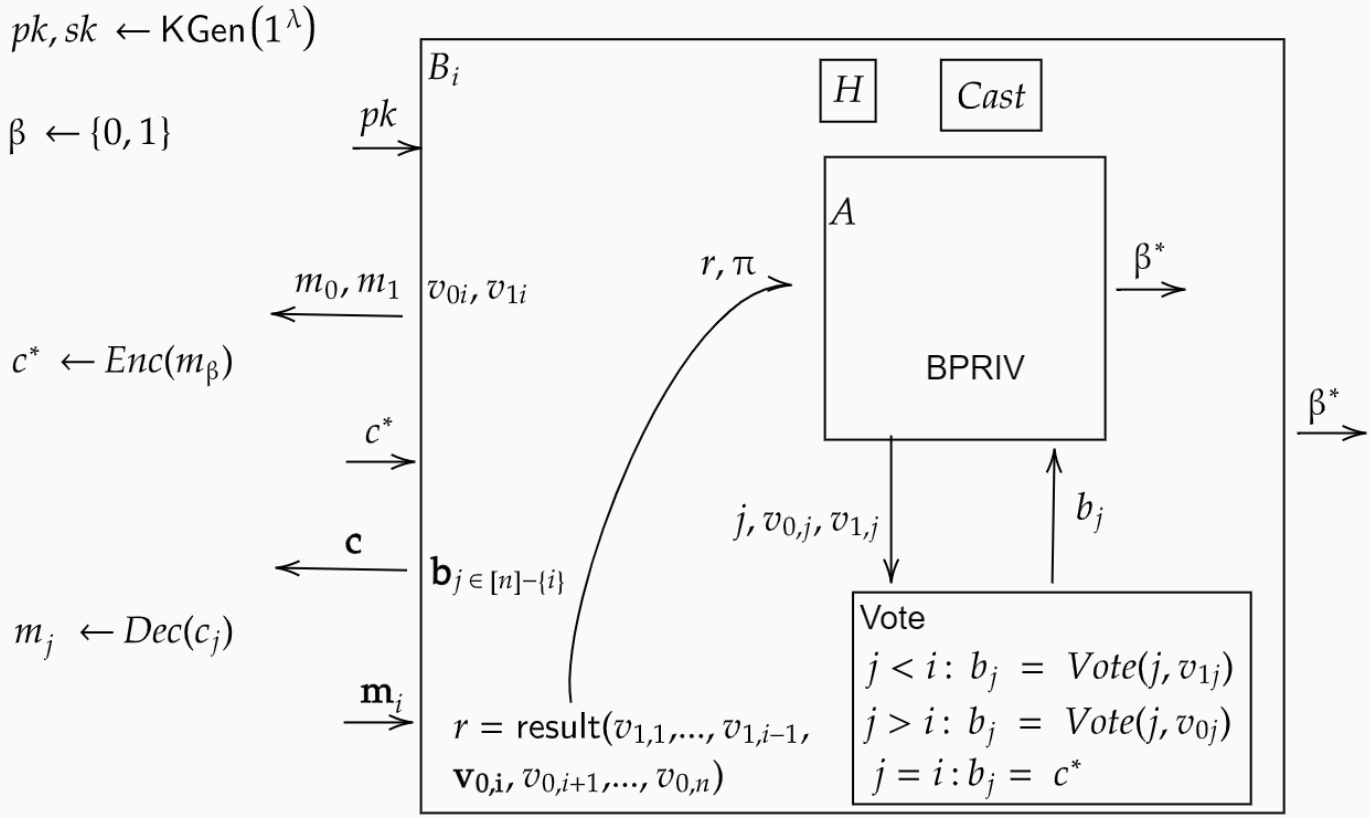
$Game_0$: BPRIV^0 - $\mathcal{A}$ sees BB_0 and the real result of the tally

$Game_1$: The tally proof in BPRIV^0 is simulated as
$\pi'_T = \text{Simulate}(g, \text{pk}, R, Sg^{-t}, c)$.
Based on the ZK property of π_T, the $\mathcal{A}$ has a negligible advantage in distinguishing between $Game_0$,$Game_1$

$\{Game_{2i}\}_{i=1}^{n_{Hon}}$
For all ballots cast using **Vote** (challenger entries $(i, b_0, vt_0, b_1, vt_1)$) replace b_{0i} in BB_0 with b_{1i} from BB_1.
$\mathcal{A}$ notices this with negligible probability due to the $NM - CPA$ property of Enc + PoK

$Game_{2n}$ is BPRIV^1

$NM - CPA \Rightarrow$ BPRIV



Everlasting Privacy

Motivation

- Ballot secrecy is provided through encryption schemes
- Protection relies on computational hardness assumptions
- What if these assumptions are broken?

- Vote contents might be useful to a future oppressive government
- The need for verifiability makes election data publicly available
- But such a regime might also use insider information
- This threat might constitute an indirect coercion attempt

Approaches i

Perfectly hiding commitment schemes

- Ballot: Perfectly hiding commitment of vote
- For counting: openings are required
- How do voters post the openings?
  - Encrypt the openings
  - Secret sharing with the tallying authorities

Practical Everlasting Privacy: Everlasting privacy towards the public

External adversary has the same view with the voters

Excludes insiders

Approaches ii

Anonymous casting (+ blind signatures)

- Disassociate identity with ballot
- Provide a blind signature to identified ballot to signal eligibility
- Identity is no longer required
- Blinded ballots cast anonymously

A further advantage: TA is not required to be trusted for privacy

Applying Everlasting Privacy to Helios i

- Pedersen commitment $\text{Commit}(m, r) = g^m h^r$ $g, h \leftarrow\!\!\$\ \mathbb{G}$
- Homomorphic:
  $\text{Commit}(m_1, r_1) \cdot \text{Commit}(m_2, r_2) = \text{Commit}(m_1 + m_2, r_1 + r_2)$
- Perfectly hiding - computationally binding
- Problem: ElGamal cannot be used for encrypting m, r
- Use a compatible cryptosystem (many approaches)

Applying Everlasting Privacy to Helios ii

Vote(i, vt_i)

$(b_i, \mathrm{ok}_i) \leftarrow \mathrm{Commit}(\mathrm{ck}, vt_i)$

$[\mathrm{vt}_i] \leftarrow \mathrm{Enc}(\mathrm{pk}_{\mathrm{TA}}, vt_i)$

$[\mathrm{ok}_i] \leftarrow \mathrm{Enc}(\mathrm{pk}_{\mathrm{TA}}, \mathrm{ok}_i)$

$\mathrm{BB} \Leftarrow b_i$

Send $[vt_i], [\mathrm{ok}_i]$ **to** TA

Tally$(\mathrm{sk}_{\mathrm{TA}})$

$$[vt] = \mathrm{Enc}(\mathrm{pk}_{\mathrm{TA}}, \sum_{i=1}^{n} vt_i) = \prod_{i=1}^{n} [vt_i]$$

$$vt = \mathrm{Dec}(\mathrm{sk}_{\mathrm{TA}}, [vt])$$

$$\mathrm{ok} = \mathrm{Enc}(\mathrm{pk}_{\mathrm{TA}}, \sum_{i=1}^{n} \mathrm{ok}_i) = \prod_{i=1}^{n} [\mathrm{ok}_i]$$

$$\mathrm{ok} = \mathrm{Dec}(\mathrm{sk}_{\mathrm{TA}}, [\mathrm{ok}])$$

$$b = \prod_{i=1}^{n} b_i$$

if $\mathrm{CS.Open}(b, vt, \mathrm{ok}) = 1$ **then**

return vt

endif

endfor

Helios Extensions for receipt-freeness and participation privacy

Intuition

Privacy does not deal with malicious *voters*, that willingly interact with an attacker before/after voting to sell their votes

Assumption: $\mathcal{A}$ does not monitor voter during vote casting. Only at distinct moments before / after.

Receipt-Freeness
A (malicious) voter cannot prove how she voted, even if she wants to

Helios is not receipt-free
The malicious voter can reveal the randomness used in ballot creation (encryption) $\mathcal{A}$ can recompute the encryption and check the BB for the ballot

Revoting does not help. $\mathcal{A}$ can check which ballot finally appears in the published BB.

Solutions

BeleniosRF

- Rerandomization Server
- Changes the randomization contained in the ballot
- The voter is not alone in contributing randomness
- So the voter has no use in revealing it

Helios-KTV

- Deniable vote updating
- The voter casts b_1 (the selled vote vt_1)
- Later updates the ballot to b'_1 (a new vote vt'_1)
- Anonymization layer required
- Revoting is not allowed in many jurisdictions

Both define receipt-freeness as an extension of BPRIV

BPRIV for receipt-freeness i

$\mathcal{A}$ has additionally access to the following oracles:

- **Register**(i): Generate credentials for the voter
- **Corrupt**(i): Corrupt a voter for receipt-freeness (get the voter's key)
- **Receipt**(b_0, b_1): The corrupted voter casts two ballots to the respective BB. They are assumed to encode information that will allow vote selling (e.g. set last bit to one)

$\mathcal{A}$ can call **Tally** only - once the rest at will

$\mathcal{A}$ must guess which BB is tallied

A ballot b_b submitted through **Receipt**(b_0, b_1) can be swapped with $b_{1-\mathrm{b}}$ without $\mathcal{A}$ noticing

BPRIV for receipt-freeness ii

Algorithm 3: BPRIV extensions for BeleniosRF

Oracle *Register*(i)
- $(\mathsf{pk}_i, \mathsf{sk}_i) \leftarrow \mathsf{Register}(i)$
- $\mathsf{V}_{El} \Leftarrow (i, \mathsf{pk}_i, \mathsf{sk}_i)$
- return pk_i

Oracle *Corrupt*(i)
- **if** $i \in$ **then**
  - $\mathsf{V}_{Corr} \Leftarrow (i, \mathsf{pk}_i, \mathsf{sk}_i)$
  - return $(\mathsf{pk}_i, \mathsf{sk}_i)$

Oracle *Receipt*(i, b_0, b_1)
- **if** $i \in \mathsf{V}_{Corr}$ **then**
  - **if** $\mathsf{Valid}(b_0, \mathsf{BB}_0)$ **AND** $\mathsf{Valid}(b_1, \mathsf{BB}_1)$ **then**
    - $\mathsf{BB}_0 \Leftarrow b_0$
    - $\mathsf{BB}_1 \Leftarrow b_1$

- The ballot b_0 can be replaced with b_1 without $\mathcal{A}$ noticing
- No strategy on behalf of the voter needs to be applied
  - More practical schemes (single pass)
  - Revoting is not always legal
- Helios is not receipt-free: $\mathcal{A}$ can encode information in $\mathsf{BB}_0, \mathsf{BB}_1$ (randomness) to distinguish them

Signatures on Randomizable Ciphertexts i

- $(\mathsf{dk}, \mathsf{pk}, \mathsf{vk}, \mathsf{sk}) \leftarrow \mathsf{KGen}(1^\lambda)$
- $\sigma \leftarrow \mathsf{Sign}(\mathsf{sk}, m, \mathsf{pk})$
- $\{0,1\} \leftarrow \textbf{Verify}(\mathsf{vk}, \sigma, m)$
- $c \leftarrow \mathsf{Enc}(\mathsf{pk}, m, \mathsf{vk})$
- $m \leftarrow \mathsf{Dec}(\mathsf{dk}, c)$

$$(c', \sigma') \leftarrow \mathsf{Random}(\mathsf{pk}, \mathsf{vk}, c, \sigma) \approx (\mathsf{Enc}(\mathsf{pk}, m, \mathsf{vk}), \mathsf{Sign}(\mathsf{sk}, c', \mathsf{pk}))) \text{ where}$$

$$c = \mathsf{Enc}(\mathsf{pk}, m, \mathsf{vk}) \sigma = \mathsf{Sign}(\mathsf{sk}, c, \mathsf{pk})$$

$$\textbf{Vote}(id, \mathsf{pk}, \mathsf{vk}, vt) = (c' = \mathsf{Enc}(\mathsf{pk}, m, \mathsf{vk}), \mathsf{Sign}(\mathsf{sk}, c', \mathsf{pk}))$$

$$\mathsf{Append}(b) = \mathsf{Random}(\mathsf{pk}, \mathsf{vk}, b)$$

Verifiability: Validate the signatures except the proofs

Helios-KTV i

Participation privacy: The voter cannot prove *if she voted or abstained*.

More serious attack for Internet voting due to larger scale

Basic idea: *dummy* ballots

- cast by a TTP - the posting trustee
- at random voting intervals during the voting phase
- contain a null vote
- many dummy ballots per voter - id $\{\textbf{VoteDummy}(i) = \textbf{Vote}(i, \mathrm{pk}, \cdot, 0)\}_{i=1}^{m}\ m \leftarrow\!\!\$\ [n]$
- before tallying: homomorphically aggregate all ballots per voter id
- the end result contains the real vote of voter id

Helios-KTV ii

For Receipt-Freeness: Deniable vote updating

- the $\mathcal{A}$ wants to cast a ballot $b_{\mathcal{A}} = \textbf{Vote}(i, \text{pk}, \text{sk}_i, vt_{\mathcal{A}})$
- the voter obeys
- at a different time (earlier or later) the voter casts a different ballot b_v that updates to correspond to vote vt_v
- $\textbf{DeniablyUpdate}(i, vt_v, vt_{\mathcal{A}}) = \textbf{Vote}(i, \text{pk}, \text{sk}_i, vt_v - vt_{\mathcal{A}})$
- need to know previous vote

Assumptions

- available public key infrastructure
- anonymous channel
- $\mathcal{A}$ does not watch voter during the complete election
- trusted posting trustee, BB, TA

Extend BPRIV using two oracles

Helios-KTV iii

Main idea: Vote swapping with abstention $\mathcal{A}$ cannot distinguish if V_{i_0} abstained and V_{i_1} voted at most k times or vice versa

Algorithm 4: BPRIV extension for KTV-Helios

Oracle $\mathit{VoteAbstain}_{\mathbb{P}}(i_0, i_1, vt_1, \cdots, vt_k)$

$m_b, m_{1-b} \leftarrow\$ \mathbb{P}$

$\{b_{i_b,i}\}_{i=1}^{m_b} \leftarrow \mathsf{VoteDummy}(i_b)$

$\{b_{i_{1-b},i}\}_{i=1}^{m_{1-b}} \leftarrow \mathsf{VoteDummy}(i_{1-b})$

$\{b_{i_b,i} \leftarrow \mathsf{Vote}(i_b, \mathsf{pk}, \mathsf{sk}_{i_b}, vt_i)\}_{i=1}^{k}$

$\{\mathsf{BB}_b \Leftarrow b_{i_b,i}\}_{i=1}^{k+m_b}$

$\{\mathsf{BB}_{1-b} \Leftarrow b_{i_{1-b},i}\}_{i=1}^{m_{1-b}}$

The adversarial advantage depends on distinguishing $k + m_b$ from m_{1-b} where $m_b, m_{1-b} \leftarrow\$ \mathbb{P}$

Helios-KTV iv

For b $= 0$ the voter sells her vote. For b $= 1$ the voter tries to fool the coercer and executes **DeniablyUpdate**

distinguishing factor - number of ballots in BB_0, BB_1 - obfuscation using **VoteDummy**

Algorithm 5: BPRIV extensions for KTV-Helios

Oracle *Receipt*$(i, vt_{\mathcal{A}}, vt_v)$

- $b_{\mathcal{A}} =$ **Vote**$(i, \mathsf{pk}, \mathsf{sk}_i, vt_{\mathcal{A}})$
- $BB_0 \Leftarrow b_{\mathcal{A}}$
- $BB_1 \Leftarrow b_{\mathcal{A}}$
- $b_v =$ **DeniablyUpdate**$(i, vt_v, vt_{\mathcal{A}})$
- $BB_1 \Leftarrow b_v$
- $BB_0 \Leftarrow$ **VoteDummy**(i)
- $BB_1 \Leftarrow$ **VoteDummy**(i)

Coercion Resistance

A stronger adversary

Active methods for attack:

- Vote for a specific candidate / randomly
- Totally abstain from voting
- Yield private keys - allow to be simulated

Passive methods for attack:

- Monitor voting system throughout election
- Same for voter except for a moment of privacy

Goal: Internet voting

Note: Coercion Resistance $\rightarrow$ Receipt-Freeness

The JCJ coercion resistance framework

Intuition

$\mathcal{A}$ will not be motivated to attack, if he cannot check if the attack succeeds

Techniques

- Multiple votes per voter
- Authentication using anonymous credentials

Registration phase: voter registers a real credential

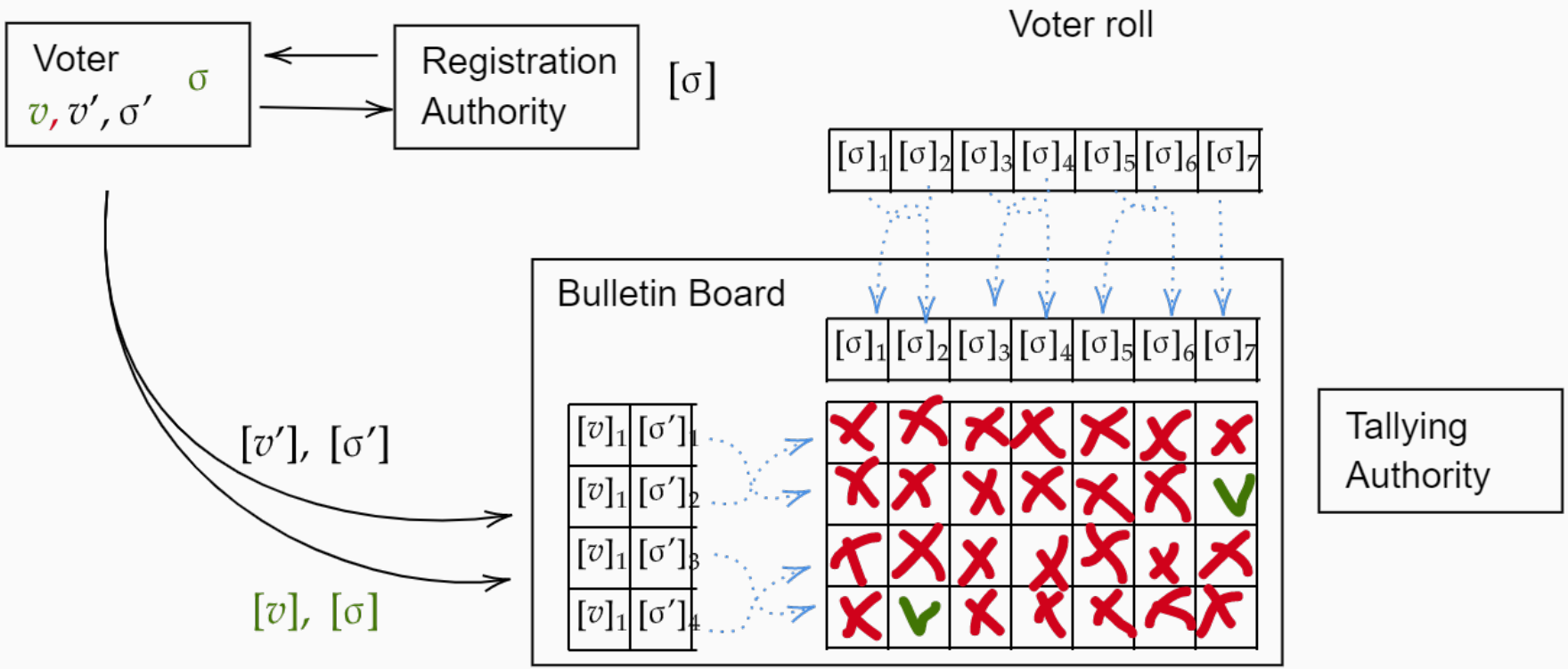
During coercion attack: voter supplies a fake credential (indistinguishable)

During moment of privacy: voter casts the real vote

During tallying: the TA must filter out fake and duplicate votes *in a verifiable manner* without disclosing which votes are not counted

How: Blind comparisons in the backend against a voter roll

The scheme



A key component: PET $(\mathsf{Enc}_{\mathsf{pk}}(m_1), \mathsf{Enc}_{\mathsf{pk}}(m_2)) = 1 \Leftrightarrow m_1 = m_2$

Plaintext Equivalence Test

Algorithm 6: PET for ElGamal ciphertexts

Input : $\mathbb{G}, g, q$, pk_i: $\prod_i^t \mathsf{pk}_i = \mathsf{pk}$, $c = (c_1, c_2)$, $c' = (c'_1, c'_2)$
Private Input: $\mathsf{sk}_i \in \mathbb{Z}_q$: $\sum_i^t \mathsf{sk}_i = \mathsf{sk}$
Output : $\{0, 1\}$

$c_{\mathsf{PET}} := \frac{c}{c'} = (\frac{c_1}{c'_1}, \frac{c_2}{c'_2})$

$z_i \leftarrow\$ \mathbb{Z}_q$

$c_{i,\mathsf{PET}} := c_{\mathsf{PET}}^{z_i} = (c_{i1}, c_{i2}) = ((\frac{c_1}{c'_1})^{z_i}, (\frac{c_2}{c'_2})^{z_i})$

$\pi_{i1} \leftarrow \mathsf{NIZK}\{(\mathbb{G}, g, q, \mathsf{pk}, c_{\mathsf{PET}}, c_{i,\mathsf{PET}}), (z_i) : c_{i,\mathsf{PET}} = c_{\mathsf{PET}}^{z_i}\}$

Publish $(c_{i,\mathsf{PET}}, \pi_{i1})$ and wait until all players have posted. Verify the proofs π_{i1} posted from other players

$\phi := \prod_i^t c_{i,\mathsf{PET}} = (\prod_i c_{1i}, \prod_i c_{2i}) = (c_{1i}^{\sum_i z_i}, c_{2i}^{\sum_i z_i}) = (x, y)$

$\psi_i := x^{sk_i}$

$\pi_{i2} \leftarrow \mathsf{NIZK}\{(\mathbb{G}, g, q, \mathsf{pk}, \psi_i), (sk_i) : \psi_i = x^{sk_i}\}$

Publish (ψ_i, π_{i2}) wait until all players have posted. Verify the proofs π_{i2} posted from other players

$\rho := y / \prod_i^t \psi_i$

return $\rho = 1$

Note: The strong Fiat-Shamir heuristic must be used If not verifiability can be broken

Assumptions

- Moment Of Privacy
- Untappable Registration (occurs one / used in multiple elections)
  - Secure transcript erasure
  - Simulation of transcript
- Anonymous casting
  - For forced abstention attack
- Coercer uncertainty about voter behavior
  - If all vote, then the abstention attack will always succeed
  - The voting authorities may inject chaffe votes on purpose

Algorithm 7: Real Coercion resistance game

$(\text{prms}, \mathsf{V}_{El}, \text{CS}) \leftarrow \textbf{Setup}(1^{\lambda})$
$\{(\text{sk}_i, \text{pk}_i) \leftarrow \text{Register}(\text{sk}_{\text{RA}}, i)\}_{i=1}^{n}$
$V_{Corr}, \leftarrow \mathcal{A}(\textbf{corrupt})$ *// Adversary corrupts voters*
$(j, vt_j) \leftarrow \mathcal{A}(V_{Hon}, \textbf{coerce})$ *// Adversary coerces voter j*
$\text{b} \leftarrow\$ \{0, 1\}$
if $\text{b} = 0$ **then**
  $\text{sk}_j^* \leftarrow \text{fakekey}(j)$ *// generate fake credential*
  $b_j \leftarrow \textbf{Vote}(j, vt_j, \text{sk}_j)$ *// Moment of privacy*
else
  $\text{sk}_j^* \leftarrow \text{sk}_j$ *// yield credential*
end
$\{b_i \leftarrow \textbf{Vote}(i, vt_i, \text{sk}_i),\}_{i=1}^{|V_{Hon}|, \mathcal{D}}$
$\text{BB} \Leftarrow \textbf{Vote}(j, vt_j, \text{sk}_j^*)$ $\{\text{BB} \Leftarrow \mathcal{A}(\text{sk}_i, vt_i, \textbf{cast})\}_{i=1}^{|V_{Corr}|}$
$(T, \pi_T, \Gamma) := \textbf{Tally}(\text{BB}, \text{sk}_{TA})$
$\text{b}' \leftarrow \mathcal{A}(T, \pi_T, \Gamma, \text{BB}, \textbf{guess})$
return $\text{b} = \text{b}'$

Algorithm 8: Ideal Coercion resistance game

$(\text{prms}, \mathsf{V}_{El}, \text{CS}) \leftarrow \textbf{Setup}(1^{\lambda})$
$\{(\text{sk}_i, \text{pk}_i) \leftarrow \text{Register}(\text{sk}_{\text{RA}}, i)\}_{i=1}^{n}$
$V_{Corr}, \leftarrow \mathcal{A}(\textbf{corrupt})$ *// Adversary corrupts voters*
$(j, vt_j) \leftarrow \mathcal{A}(V_{Hon}, \textbf{coerce})$ *// Adversary coerces voter j*
$\text{b} \leftarrow\$ \{0, 1\}$
if $\text{b} = 0$ **then**
  $b_j \leftarrow \textbf{Vote}(j, vt_j, \text{sk}_j)$
end
// Moment of privacy
$\text{sk}_j^* \leftarrow \text{sk}_j$ *// always yield credential*
$\{b_i \leftarrow \textbf{Vote}(i, vt_i, \text{sk}_i),\}_{i=1}^{|V_{Hon}|, \mathcal{D}}$
$\text{BB} \Leftarrow \textbf{Vote}(j, vt_j, \text{sk}_j^*)$
$\{\text{BB} \Leftarrow \mathcal{A}(\text{sk}_i, vt_i, \textbf{cast})\}_{i=1}^{|V_{Corr}|}$
$T := \text{ideal_tally}(\text{BB}, \text{sk}_{TA})$
$\text{b}' \leftarrow \mathcal{A}(T, \textbf{guess})$
return $\text{b} = \text{b}'$

The model iii

Why the ideal experiment?

- An alternative: Distinguish between $b = 0$ and $b = 1$?
- The tally might help the coercer distinguish if the coercion attempt succeeded
- For instance: The voter instruct a vote for 'Alice' but no 'Alice' votes are found (invariant of the protocol)
- Ideal tallying functionality ideal_tally:
  - ballots cast by V_{Hon} are treated specially
  - For ballots cast by $\mathcal{A}$:
  - performs weeding of double votes based on the extracted credential
  - If $b = 0$ votes with sk_j^* are *not* counted
  - If $b = 1$ votes with sk_j^* are counted

JCJ evolution

Main drawback Quadratic tallying complexity: $\mathcal{O}(v^2) + \mathcal{O}(nv)$ for duplicate detection and tallying

Goal $\mathcal{O}(n + v)$

3 approaches for better efficiency:

Anonymity sets
The ballot contains: The real credential

- The current credential
- The real credential from the voter roll (rerandomized)
- Some $\beta - 1$ other random credentials from the BB
- The PET takes place only among the credentials of the ballot

JCJ evolution (cont'd)

Blinded hashing

- The credential $[\sigma]$ is blinded to obtain $[\sigma]^z$
- The value z is jointly computed by the RA members
- Instead of PET $[\sigma]^z$ is decrypted to obtain σ^z (credential fingerprint)
- This fingerprint is sent through a hashtable $\{(\sigma^z, b)\}$
- For verifiability: σ^z is made public so that everybody can check the hashtable

Vulnerable to the tagging attack (Pfitzmann)

- $\mathcal{A}$ forces a voter to reveal the credential
- Then $\mathcal{A}$ posts two ballot one with $[\sigma]$ and one with $[\sigma^2]$
- Two fingerprints will be produced σ^z and one with σ^{2z}
- $\mathcal{A}$ squares all elements in the BB.
- If a square matches another element and both have been discarded then $\mathcal{A}$ learns that σ was fake

Note: This attack applies only on fake credentials, not on duplicates
So $\mathcal{O}(v^2)$ becomes $\mathcal{O}(v)$

JCJ evolution (cont'd)

Structured credentials

Check of validity is self contained in the credential

- A credential is a tuple $(r \leftarrow_{\$} \mathbb{Z}_q, a \leftarrow_{\$} \mathbb{G}, b = a^y, c = a^{x+rxy})$ where $x, y \leftarrow_{\$} \mathbb{Z}_q$ are secret keys of the RA
- r should be kept secret by the voter
- A fake credential can be made by selecting a new r
- If (r, a, b, c) is valid then (r, a^l, b^l, c^l) is valid
- $b = ([\boldsymbol{vt}], [a], [a^r], [b^r], [c], o^r) = (\cdot, u, v, w, z, \cdot)$
- Duplicates are identified by o^r
- Ballot validity if $z \cdot u^{-x} \cdot w^{-x}$ decrypts to 1

Decentralized voting

Introduction

2 Voting paradigms

- Large scale elections
  - Involvement of authorities (mixing / tallying)
  - Trust required for some properties
  - Each voters plays a small part (vote & go)
  - Existence of BB: contains all voting public data (broadcast channel with memory)
- Small(er) scale elections (boardroom)
  - Conducted by the voters themselves
  - No entity plays a special part
  - Robustness is more important: A voter cannot disrupt the election
  - Private channels lead to disputes

Can these two paradigms be combined?

Desirable properties for decentralized elections - [KY02]

- Self-Tallying (open tallying)
  Any (external) entity should be able to count the ballots
  (Implies verifiability) it does not depend on secret keys
- Perfect Ballot Secrecy
  The partial election result for a particular subset of voters can be obtained only by a coalition that contains the rest of the voters
- Dispute-Freeness Embedded mechanisms avert disputes and make the participants follow the protocol (accountability)

Can these properties be achieved while minimizing communication complexity and voter-to-voter interaction

Relationships

Theorem
Self-tallying is incompatible with robustness and privacy (at the same time)

Proof.
Assume: Self-tallying and robust

Assume: $n - 1$ from n voters show up

By self tallying: Anyone can compute the result

But: The same computation can take place even if all voters show up But this reveals the preference of the last (any) voter □

Theorem
Robustness through threshold secret sharing is incompatible with perfect ballot secrecy

Proof.
For perfect ballot secrecy the threshold must be set to n

But this is not robust □

A solution: Decentralized voting with a BB (can be replaced with a blockchain)

Anonymous Voting by 2-Round Public Discussion - [HRZ10]

Preparation
Select a group $\mathbb{G} = g$

Each voter has an identity V_i

Selects $a_i \leftarrow\!\!\$\ \mathbb{Z}_q$

Round 1 - Commitment
Each V_i posts g^{a_i}, NIZK$\{(a_i), g^{a_i}\}$

When this phase ends:
$g^{y_i} = \prod_{j=1}^{i-1} g^{a_i} / \prod_{j=i+1}^{n} g^{a_i}$

Round 2 - Voting
Each V_i selects $vt_i \in \{0, 1\}$ and posts $(g^{y_i})^{a_i} g^{vt_i}$ for some unknown $y_i \in \mathbb{Z}_q$

Round 3 - Self-tallying
Everyone computes
$\prod_{i=1}^{n} (g^{y_i})^{a_i} g^{vt_i} = \prod_{i=1}^{n} g^{vt_i} = g^{\sum_{i=1}^{n} vt_i}$

Solve a simple DLP

Analysis [HRZ10]

Correctness

$$\sum_{i=1}^{n} a_i y_i = \sum_{i=1}^{n} \sum_{j<i} a_i a_j - \sum_{i=1}^{n} \sum_{j>i} a_i a_j = 0 \text{ since: } y_i = \sum_{j<i} a_j - \sum_{j>i} a_j$$

	x_1	x_2	x_3	x_4	x_5
x_1		−	−	−	−
x_2	+		−	−	−
x_3	+	+		−	−
x_4	+	+	+		−
x_5	+	+	+	+	

Problems

Robustness: If someone refuses to vote then the result cannot be computed

Fairness: The last voter can learn the result

For multiple candidates

- Parallel voting (reject/accept) $0, 1$ for k candidates
- Baudron [BFPPS01] counters:
  - Select m st: $2^m > n$
  - To vote for candidate i cast: $\boldsymbol{vt}_i = 2^{(i-1)m}$
  - During tallying: $\sum_i \boldsymbol{vt}_i = 2^0 c_1 + 2^m c_2 + \cdots + 2^{(k-1)m} c_k$
  - Retrieve c_i

Improvements for robustness and fairness [KSRH12]

Robustness
Assume that only voters in $\mathcal{L}$ have voted.

Recovery Round
Each $V_i \in \mathcal{L}$ compute:

$$\hat{h}_i = \frac{\prod_{j \in \{i+1,n\} \setminus L} g^{a_j}}{\prod_{j \in \{1,i-1\} \setminus L} g^{a_j}}$$

and posts cancellation tokes $\hat{h}_i^{a_i}$, NIZK$\{(a_i) : log_g g^{a_i} = log_{\hat{h}} \hat{h}^{a_i}\}$

Tallying becomes: $V = \prod_{i=1}^{n} b_i = \prod_{i \in \mathcal{L}} \hat{h}_i^{a_i} (g^{y_i})^{a_i} g^{v_i} = g^{\sum_{i=1} v_i}$

No	First round	Second round	Third round	Recovery
1	g^{x_1}	commitment	$g^{x_1 y_1} = g^{x_1(-x_2-x_3-x_4-x_5)}$	$\hat{h}_1^{x_1} = g^{x_1(x_2+x_4)}$
2	g^{x_2}	commitment	Abort	-
3	g^{x_3}	commitment	$g^{x_3 y_3} = g^{x_3(x_1+x_2-x_4-x_5)}$	$\hat{h}_3^{x_3} = g^{x_3(x_4-x_2)}$
4	g^{x_4}	commitment	Abort	-
5	g^{x_5}	commitment	$g^{x_5 y_5} = g^{x_5(x_1+x_2+x_3+x_4)}$	$\hat{h}_5^{x_5} = g^{x_5(-x_2-x_4)}$

Open Vote Network [MSH17]

- Implementation of [HRZ10] using Ethereum
- Smart contracts (voting, registration, tallying)
- Voters are authenticated with their Ethereum user-controlled accounts
- Ethereum restrictions:
  - integers of 256 bits
  - expensive cryptographic computations
  - one vote or six registrations per block
  - small number of allowed local variables
  - order of transactions in a block and timers
- Maximum number of voters: 50 (due to gas limit)
  - Linear number of operations for Tally and Voter List
- Cost per voter: 0.73$

Scalable Open Vote Network [SGY20]

Improvements

- Organize voters in Merkle Tree only the root is stored (256 bits)
- Instead of voter list a voter provides a proof of membership
- Tally off-chain by an untrusted tallier
  - Publish computation trace in Merkle Tree ((i, t_i))
  - Subject to verification

```
def TallyVotes(c: array[]):
    t = 1
    for i=1 to n:
        t = Mul(c[i], t)
    return t
```

Step i	c_i	t_{i-1}	t_i
1	c_1	$t_0 = 1$	$t_1 = c_1$
2	c_2	t_1	$t_2 = c_2 \cdot t_1$
.	.	.	...
n	c_n	t_{n-1}	$t_n = c_n \cdot t_{n-1}$

Blockchain and voting i

Conceptual similarity between blockchain and the BB

- Append-only
- Broadcast channel
- No central authority - anyone can be a miner (given enough computing power)
- Pseudonymity

Good for universal/individual verifiability (recorded as cast)

But...

- Registration/authentication/eligibility verifiability are inherently centralized
- Does not help with verifying voter intent
- Does not help with coercion-resistance / receipt-freeness

Blockchain and voting ii

- Intensifies threats associated with everlasting privacy
- Is it actually decentralized? (concentration of mining power)
- Scalability issues

To sum up... using Blockchain for voting solves a *small* part of the problem with an unnecessarily big hammer (Ben Adida, 2017)

However... it might be useful for different types of elections - new election paradigms on a smaller scale with many different permission-types of blockchains

Relations between properties

Verifiability i

Orthogonal properties

IV without UV

- Construct a scheme with $(vt_i, r_i) \leftarrow \textbf{Vote}(i, vt_i)$
- Individual verifiability because of r_i
- Allows ballots stuffing
- BeleniosRF: RA and rerandomization server collusion can change contents of vote

UV without IV if everyone votes

- Construct a scheme with $vt_i \leftarrow \textbf{Vote}(i, vt_i)$
- Clash for same preferences

Verifiability ii

EV implies IV

- Nobody can construct a ballot unless they know sk_i (private voter credential)
- sk_i allows individual verifiability

Verifiability and Privacy i

Universal verifibiality is uncompatible with unconditional privacy

- Unconstrained $\mathcal{A}$
- Compute election tally of a sublist of $n - 1$ voters
- Decryption and comparison with the tally of n voters
- Reveal the option of the remaining voter

Verifiability and Privacy ii

Privacy implies individual verifiability

- Assume no individual verifiability
- A corrupt EA can replace all ballots except one
- Learn the particular preference

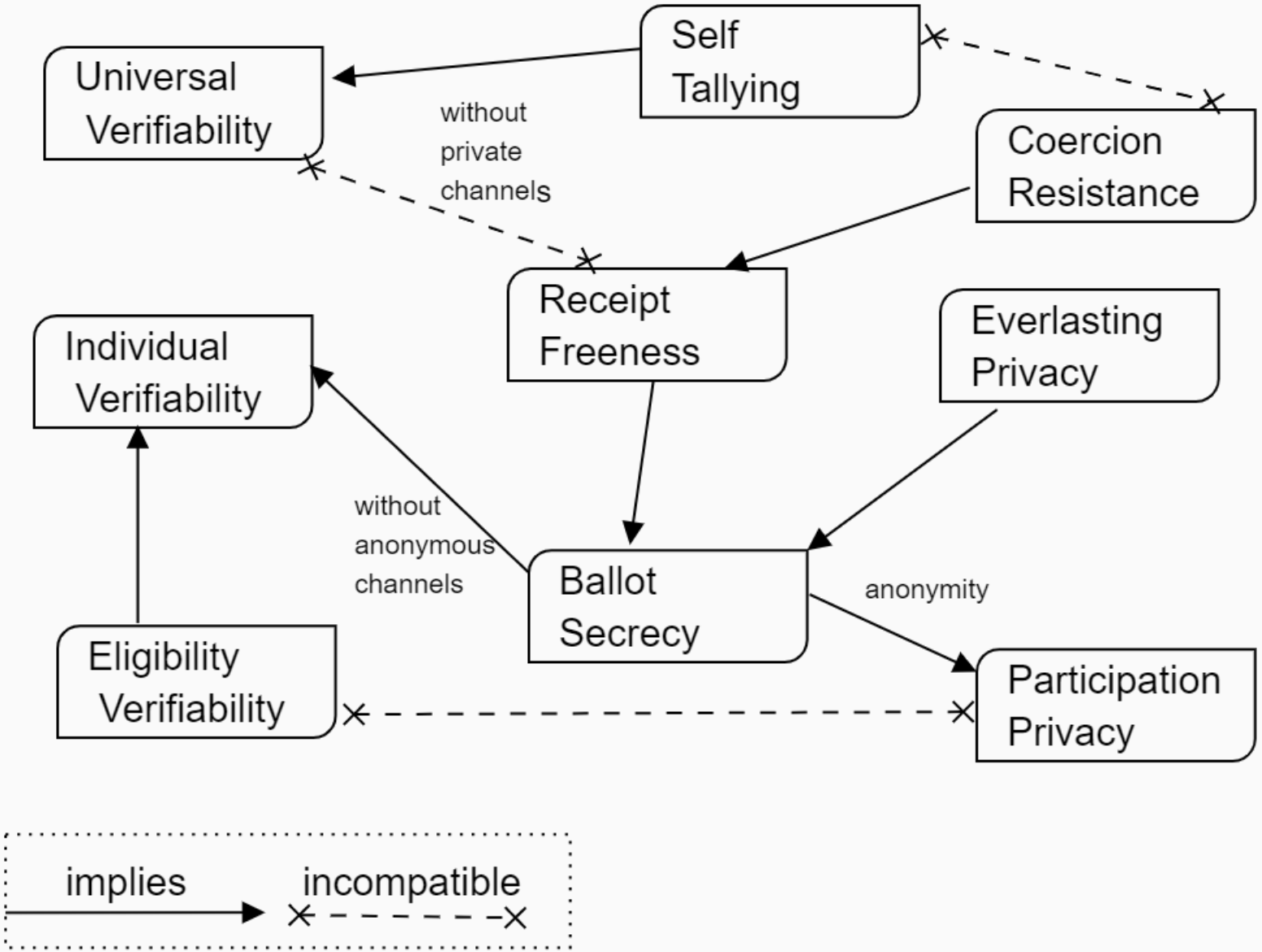
Receipt-freeness is incompatible with universal/individual verifiability

- Encryption randomness can be used as a receipt
- Without randomness: receipt-freeness
- With randomness: verifiability
- No private or anonymous channels are used

Privacy

- Receipt freeness implies privacy
- Everlasting privacy implies privacy
- The reverse does not hold
- Everlasting privacy is related to coercion resistance
- Coercion resistance and receipt-freeness: Depends on the model

Overview of relationships



Open questions

- Security Analysis of Zeus
  - Check if the pitfalls of the Fiat - Shamir Heuristic apply
  - Formal proof of verifiability and privacy
  - Security issues
- Decentralized voting with coercion resistance
  - Ring signatures
  - Self-tallying incompatibility
- Relations between properties
  - Formal proofs

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