

# Dynamic Epistemic Logic (ALMA Spring 2021)

## Final Take Home Exam

**Deadline:** Thursday July 1st, 2021 at 23:55

All the references below refer to our sources textbook “Dynamic Epistemic Logic”.

### Exercise 1

- (a) Show that the modal formula  $\Box\Box p \rightarrow \Box p$  defines the property *weak density*  $\forall s\forall t(sRt \rightarrow \exists u(sRu \wedge uRt))$ .
- (b) Write a modal formula that defines the property  $\forall x\forall y(xRy \rightarrow xRx)$ . Justify your answer.

### Exercise 2

The aces and eights game is a simple game that involves some sophisticated reasoning about knowledge. It is played with a deck consisting of just four aces and four eights. There are three players. Six cards are dealt out, two to each player. The remaining two cards are left face down. Without looking at the cards, each of the players raises them up to his or her forehead, so that the other two players can see them but he or she cannot. Then all of the players take turns trying to determine which cards they’re holding (note that for our version of the game the suit of a card doesn’t matter). If a player does not know which cards he or she is holding, the player must say so. Suppose Alice, Bob and you are playing the game. Of course, it is common knowledge that none of you would ever lie and that you are all perfect reasoners.

- (a) In the first game, Alice, who goes first, holds two aces, and Bob, who goes second, holds two eights. Both Alice and Bob say that they cannot determine what cards they are holding. What cards are you holding?
- (b) In the second game, you go first. Alice, who goes second holds two eights. Bob who goes third, holds an ace and an eight. No one is able to determine what she or he is holding at her or his first turn. What are you holding?

- (c) In the third game, you go second. Alice who goes first, holds an ace and an eight. Bob, who goes third, also holds an ace and an eight. No one is able to determine what she or he is holding at her or his first turn. Alice cannot determine her cards at her second turn either. What are you holding?
- (d) How many possible worlds are there for this game? Recall that the suit of a card doesn't matter.
- (e) Draw the Kripke structure describing the game.
- (f) Consider the situation described in part (a). Which edges disappear from the structure when you hear that Alice and Bob cannot determine which cards they have?

### **Exercise 3**

Solve exercises 4.20 and 4.21. For exercise 4.21 the model of example 1 is on page 68.

### **Exercise 4**

Solve exercise 4.48.

*Hint:* You will need table 4.1 on page 89. Remember that table 4.1 is not complete! The rule “from  $\phi$  follows  $[\psi]\phi$ ” is missing from this table. Use the previous rule freely in your solution.

### **Exercise 5**

Solve exercise 6.28.

### **Exercise 6**

Solve the first bullet from exercise 6.39.