

Belief Revision

Konstantakatos Vangelis

National Technical University of Athens

Outline

Motivation

History

The AGM Approach

Expansion

Contraction

Revision

Characterisation Results

References

Motivation

(Van Benthem)

“Rational agents are not those who are always correct, but those who have an ability for the dynamics of correction.”

“Beliefs come with belief revision, an ability to correct ourselves when contradicted by the facts.”

Motivation

- ▶ In Dynamic Epistemic Logic we seek to study how agent's knowledge and beliefs change over time.
- ▶ A related, but more philosophically oriented area of research, is that of Belief Revision.
- ▶ Belief Revision studies what happens when an agent is confronted with new facts, that go against his prior beliefs
- ▶ Belief Revision and DEL are concerned with similar subjects, but the approach and the focus are different.
- ▶ Belief Revision has traditionally restricted attention to single agent, "ontic" belief change.
- ▶ While DEL focuses on the multiagent setting, and is concerned with beliefs about beliefs.

Motivation

- ▶ However, at least originally, DEL was designed to model only cases in which the newly received information is consistent with the agents prior doxastic or epistemic state.
- ▶ More recently, the focus of DEL has been to combine some ideas from Belief Revision, to deal with scenarios in which agents receive surprising new information that contradict what they already believe.

History

- ▶ Belief Revision is the child of two research traditions, philosophy and computer science.
- ▶ Belief change has been the subject of philosophy since antiquity. Belief Revision can be seen as an attempt to formalise Epistemology.
- ▶ In computer science, we have databases and we need rules to update them.
- ▶ Furthermore, with the emergence of artificial intelligence, we need to design rational agents.

History

- ▶ In 1985 the paper, "On the Logic of Theory Change: Partial Meet Contraction and Revision Functions" was the seminal work that sparked the birth of Belief Revision.
- ▶ It was written by Carlos Alchourron(1931-1996), Peter Gärdenfors(1949) and David Makinson(1941), hence called the AGM approach to belief revision.
- ▶ Alchourron and Makinson were previously researching about derogations of legal systems, and the hierarchies of regulations and their logic.
- ▶ Gärdenfors was into philosophy of science and counterfactual reasoning.

Setting

- ▶ Beliefs are propositional formulas.
- ▶ Reasoning about change is done on a meta level.

Setting

- ▶ Beliefs are propositional formulas.
- ▶ Reasoning about change is done on a meta level.
- ▶ We are not in a modal logic setting! $B\phi$ is not in the object language, instead we say that $\phi \in \mathcal{K}$ where \mathcal{K} is a belief set.

Belief Sets

Definition 1

We define \mathcal{L}_0 to be the set of propositional formulas, generated by some set of atoms \mathbb{P} and the classical connectives. Let

$Cn(\Sigma) = \{\sigma \in \mathcal{L}_0 \mid \Sigma \vdash \sigma\}$ be the classical consequence operator. A belief set \mathcal{K} is a set of propositional formulas closed under $Cn()$, i.e. $Cn(\mathcal{K}) = \mathcal{K}$.

Belief Sets

Levi

“A belief set consists of the sentences that someone is committed to believe, not those that she actually believes in”

Belief Sets

Ex.1

if $p, q \in \mathcal{K}$ then $p \wedge q, p \vee q, p \rightarrow q, q \rightarrow p, p \leftrightarrow q \in \mathcal{K}$

Belief Sets

Ex.1

if $p, q \in \mathcal{K}$ then $p \wedge q, p \vee q, p \rightarrow q, q \rightarrow p, p \leftrightarrow q \in \mathcal{K}$

Ex.2

A belief set \mathcal{K} cannot be empty. It will always contain at least all of the propositional tautologies!

Belief Sets

Ex.1

if $p, q \in \mathcal{K}$ then $p \wedge q, p \vee q, p \rightarrow q, q \rightarrow p, p \leftrightarrow q \in \mathcal{K}$

Ex.2

A belief set \mathcal{K} cannot be empty. It will always contain at least all of the propositional tautologies!

Ex.3

A belief set \mathcal{K} can be inconsistent. However because it is closed under classical consequence, even adding a single inconsistency will lead to \mathcal{K} containing every single formula in the language. We denote this unique inconsistent belief set \mathcal{K}_\perp .

Types of Change

The AGM approach distinguishes three types of change given a belief set \mathcal{K} and some new information ϕ .

- ▶ Expansion of the belief set \mathcal{K} by a formula ϕ , denoted $\mathcal{K} \oplus \phi$, means accepting ϕ , even if it yields an inconsistency.
- ▶ Contraction of the belief set \mathcal{K} by a formula ϕ , denoted $\mathcal{K} \ominus \phi$ means removing ϕ and everything that implies ϕ from the belief set.
- ▶ Revision of the belief set \mathcal{K} by a formula ϕ , denoted $\mathcal{K} \circledast \phi$ means incorporating ϕ into a new consistent belief set, that is otherwise as "similar" as possible to \mathcal{K}

Types of Change

The AGM approach distinguishes three types of change given a belief set \mathcal{K} and some new information ϕ .

- ▶ Expansion of the belief set \mathcal{K} by a formula ϕ , denoted $\mathcal{K} \oplus \phi$, means accepting ϕ , even if it yields an inconsistency.
- ▶ Contraction of the belief set \mathcal{K} by a formula ϕ , denoted $\mathcal{K} \ominus \phi$ means removing ϕ and everything that implies ϕ from the belief set.
- ▶ Revision of the belief set \mathcal{K} by a formula ϕ , denoted $\mathcal{K} \circledast \phi$ means incorporating ϕ into a new consistent belief set, that is otherwise as "similar" as possible to \mathcal{K}

Remark: Deliberation about the status of ϕ is not part of these processes. It is already somehow been decided what to do with ϕ . In a way the source of the information is considered to be absolutely trustworthy.

Postulates

- ▶ The AGM theory, provides us with a set of postulates, i.e. a series of "rationality conditions" that are meant to precisely govern the way in which a rational agent should revise his beliefs.
- ▶ The guiding principle is that of minimal change or information economy. Don't add or remove anything from the belief set, except what you absolutely have to.

Expansion

Expansion is in a way the simplest of the three types of change. We will see that it has a unique and simple characterisation, and coming up with postulates may seem unnecessary. In fact they weren't in the first version of AGM. However we will provide the postulates in order to have a uniform approach for all the operators and to make some interesting properties of expansion explicit.

Expansion Postulates

- $(\mathcal{K} \oplus 1)$ $\mathcal{K} \oplus \phi$ is a belief set *type*
- $(\mathcal{K} \oplus 2)$ $\phi \in \mathcal{K} \oplus \phi$ *success*
- $(\mathcal{K} \oplus 3)$ $\mathcal{K} \subseteq \mathcal{K} \oplus \phi$ *expansion*
- $(\mathcal{K} \oplus 4)$ if $\phi \in \mathcal{K}$ then $\mathcal{K} = \mathcal{K} \oplus \phi$ *minimal action*
- $(\mathcal{K} \oplus 5)$ For all \mathcal{H} , if $\mathcal{K} \subseteq \mathcal{H}$ then $\mathcal{K} \oplus \phi \subseteq \mathcal{H} \oplus \phi$ *monotony*
- $(\mathcal{K} \oplus 6)$ $\mathcal{K} \oplus \phi$ is the smallest set satisfying 1 – 6 *minimal change*

Expansion Postulates

($\mathcal{K} \oplus 1$) $\mathcal{K} \oplus \phi$ is a belief set *type*

Postulate *type* guarantees that after expansion we end up with a belief set, rather than for example \emptyset , or a set of belief sets, or a set not closed under C_n .

Expansion Postulates

$(\mathcal{K} \oplus 1)$ $\mathcal{K} \oplus \phi$ is a belief set *type*

Postulate *type* guarantees that after expansion we end up with a belief set, rather than for example \emptyset , or a set of belief sets, or a set not closed under Cn .

$(\mathcal{K} \oplus 2)$ $\phi \in \mathcal{K} \oplus \phi$ *success*

Once we have decided to accept new information, it should be incorporated into our beliefs.

Expansion Postulates

$$(\mathcal{K} \oplus 3) \mathcal{K} \subseteq \mathcal{K} \oplus \phi$$

Don't remove anything.

expansion

Expansion Postulates

$$(\mathcal{K} \oplus 3) \quad \mathcal{K} \subseteq \mathcal{K} \oplus \phi$$

Don't remove anything.

expansion

Postulates 1-3 guarantee minimality from below: nothing should be given up when expanding.

Expansion Postulates

$(\mathcal{K} \oplus 4)$ if $\phi \in \mathcal{K}$ then $\mathcal{K} = \mathcal{K} \oplus \phi$

Don't do anything if you already believe ϕ .

minimal action

Expansion Postulates

$(\mathcal{K} \oplus 4)$ if $\phi \in \mathcal{K}$ then $\mathcal{K} = \mathcal{K} \oplus \phi$

Don't do anything if you already believe ϕ .

minimal action

$(\mathcal{K} \oplus 5)$ For all \mathcal{H} , if $\mathcal{K} \subseteq \mathcal{H}$ then $\mathcal{K} \oplus \phi \subseteq \mathcal{H} \oplus \phi$

This is similar to 4. It can be derived by the others, but is given for historical reasons.

monotony

Expansion Postulates

$(\mathcal{K} \oplus 4)$ if $\phi \in \mathcal{K}$ then $\mathcal{K} = \mathcal{K} \oplus \phi$ *minimal action*
Don't do anything if you already believe ϕ .

$(\mathcal{K} \oplus 5)$ For all \mathcal{H} , if $\mathcal{K} \subseteq \mathcal{H}$ then $\mathcal{K} \oplus \phi \subseteq \mathcal{H} \oplus \phi$ *monotony*
This is similar to 4. It can be derived by the others, but is given for historical reasons.

$(\mathcal{K} \oplus 6)$ $\mathcal{K} \oplus \phi$ is the smallest set satisfying 1 – 6 *minimal change*
Don't add more than absolutely necessary.

Characterisation of Expansion

Theorem 2

A function \oplus satisfies $(\mathcal{K} \oplus 1) - (\mathcal{K} \oplus 6)$ iff $\mathcal{K} \oplus \phi = \text{Cn}(\mathcal{K} \cup \{\phi\})$

Contraction

Contraction is the act of giving up a belief. This is not as straightforward as expansion. For example suppose $\mathcal{K} = \text{Cn}(p, q, r)$ and we want to give up the belief of q . Simply removing q from the belief set does nothing, since $p, p \rightarrow q \in \text{Cn}(p, q, r)$, and q would immediately be reinserted in the belief set.

Contraction

Contraction is the act of giving up a belief. This is not as straightforward as expansion. For example suppose $\mathcal{K} = \text{Cn}(p, q, r)$ and we want to give up the belief of q . Simply removing q from the belief set does nothing, since $p, p \rightarrow q \in \text{Cn}(p, q, r)$, and q would immediately be reinserted in the belief set.

Therefore we also have to remove everything the implies q

Contraction

What if we wanted to contract with $p \wedge q$?

Contraction

What if we wanted to contract with $p \wedge q$?

Obviously there seems to be no reason to remove r but under the principle of informational economy, we should give up only one of p and q . It is not clear how to make such a choice!

In the following postulates there is an underlying assumption that the result of a contraction is unique, however they do not give us a way to make such a choice.

Contraction

What if we wanted to contract with $p \wedge q$?

Obviously there seems to be no reason to remove r but under the principle of informational economy, we should give up only one of p and q . It is not clear how to make such a choice!

In the following postulates there is an underlying assumption that the result of a contraction is unique, however they do not give us a way to make such a choice.

We will have to later on bring in "extra-logical" components, to uniquely define contraction.

Contraction Postulates

- ($\mathcal{K} \ominus 1$) $\mathcal{K} \ominus \phi$ is a belief set *type*
- ($\mathcal{K} \ominus 2$) $\mathcal{K} \ominus \phi \subseteq \mathcal{K}$ *contraction*
- ($\mathcal{K} \ominus 3$) if $\phi \notin \mathcal{K}$ then $\mathcal{K} = \mathcal{K} \ominus \phi$ *minimal action*
- ($\mathcal{K} \ominus 4$) if $\not\vdash \phi$ then $\phi \notin \mathcal{K} \ominus \phi$ *success*
- ($\mathcal{K} \ominus 5$) If $\phi \in \mathcal{K}$ then $\mathcal{K} \subseteq (\mathcal{K} \ominus \phi) \oplus \phi$ *recovery*
- ($\mathcal{K} \ominus 6$) If $\vdash \phi \leftrightarrow \psi$ then $\mathcal{K} \ominus \phi = \mathcal{K} \ominus \psi$ *extensionality*
- ($\mathcal{K} \ominus 7$) $((\mathcal{K} \ominus \phi) \cap (\mathcal{K} \ominus \psi)) \subseteq \mathcal{K} \ominus (\phi \wedge \psi)$ *min-conjunction*
- ($\mathcal{K} \ominus 8$) if $\phi \notin \mathcal{K} \ominus (\phi \wedge \psi)$ then $\mathcal{K} \ominus (\phi \wedge \psi) \subseteq \mathcal{K} \ominus \phi$ *max-conjunction*

Contraction Postulates

($\mathcal{K} \ominus 1$) $\mathcal{K} \ominus \phi$ is a belief set *type*
Postulate *type* guarantees that after contraction we end up with a belief set, rather than for example \emptyset , or a set of belief sets, or a set not closed under C_n .

Contraction Postulates

$(\mathcal{K} \ominus 1)$ $\mathcal{K} \ominus \phi$ is a belief set *type*

Postulate *type* guarantees that after contraction we end up with a belief set, rather than for example \emptyset , or a set of belief sets, or a set not closed under C_n .

$(\mathcal{K} \ominus 2)$ $\mathcal{K} \ominus \phi \subseteq \mathcal{K}$ *contraction*

Nothing should be added.

Contraction Postulates

$(\mathcal{K} \ominus 1)$ $\mathcal{K} \ominus \phi$ is a belief set *type*
Postulate *type* guarantees that after contraction we end up with a belief set, rather than for example \emptyset , or a set of belief sets, or a set not closed under Cn .

$(\mathcal{K} \ominus 2)$ $\mathcal{K} \ominus \phi \subseteq \mathcal{K}$ *contraction*
Nothing should be added.

$(\mathcal{K} \ominus 3)$ if $\phi \notin \mathcal{K}$ then $\mathcal{K} = \mathcal{K} \ominus \phi$ *minimal action*
Don't do anything to stop believing something that you already did not believe.

Contraction Postulates

$(\mathcal{K} \ominus 4)$ if $\phi \not\vdash \mathcal{K}$ then $\phi \notin \mathcal{K} \ominus \phi$ *success*

Unless you are trying to stop believing in a tautology, you should succeed.

Contraction Postulates

$(\mathcal{K} \ominus 4)$ if $\phi \not\vdash \mathcal{K}$ then $\phi \notin \mathcal{K} \ominus \phi$ *success*

Unless you are trying to stop believing in a tautology, you should succeed.

$(\mathcal{K} \ominus 5)$ If $\phi \in \mathcal{K}$ then $\mathcal{K} \subseteq (\mathcal{K} \ominus \phi) \oplus \phi$ *recovery*

This is motivated by the principle of minimal change, i.e. remove as little as possible. So much is retained after removal of ϕ , that everything will be reincluded if we add ϕ again.

Contraction Postulates

$(\mathcal{K} \ominus 4)$ if $\phi \not\vdash \mathcal{K}$ then $\phi \notin \mathcal{K} \ominus \phi$ *success*

Unless you are trying to stop believing in a tautology, you should succeed.

$(\mathcal{K} \ominus 5)$ If $\phi \in \mathcal{K}$ then $\mathcal{K} \subseteq (\mathcal{K} \ominus \phi) \oplus \phi$ *recovery*

This is motivated by the principle of minimal change, i.e. remove as little as possible. So much is retained after removal of ϕ , that everything will be reincluded if we add ϕ again.

It is however a very controversial postulate.

Contraction Postulates

$(\mathcal{K} \ominus 4)$ if $\phi \not\vdash \mathcal{K}$ then $\phi \notin \mathcal{K} \ominus \phi$ *success*

Unless you are trying to stop believing in a tautology, you should succeed.

$(\mathcal{K} \ominus 5)$ If $\phi \in \mathcal{K}$ then $\mathcal{K} \subseteq (\mathcal{K} \ominus \phi) \oplus \phi$ *recovery*

This is motivated by the principle of minimal change, i.e. remove as little as possible. So much is retained after removal of ϕ , that everything will be reincluded if we add ϕ again.

It is however a very controversial postulate.

$(\mathcal{K} \ominus 6)$ If $\vdash \phi \leftrightarrow \psi$ then $\mathcal{K} \ominus \phi = \mathcal{K} \ominus \psi$ *extensionality*

The result of the contraction should not depend on the syntactic representation.

Contraction Postulates

$(\mathcal{K} \ominus 7) ((\mathcal{K} \ominus \phi) \cap (\mathcal{K} \ominus \psi)) \subseteq \mathcal{K} \ominus (\phi \wedge \psi)$ *min-conjunction*

$(\mathcal{K} \ominus 8)$ if $\phi \notin \mathcal{K} \ominus (\phi \wedge \psi)$ then $\mathcal{K} \ominus (\phi \wedge \psi) \subseteq \mathcal{K} \ominus \phi$
max-conjunction

These postulates give constraints on the behaviour of \ominus when contracting with a conjunction.

Theorem 3

Let \ominus satisfy 1-6. Then it satisfies 7-8 iff we have for any ϕ, ψ one of the following:

1. $\mathcal{K} \ominus (\phi \wedge \psi) = \mathcal{K} \ominus \phi$
2. $\mathcal{K} \ominus (\phi \wedge \psi) = \mathcal{K} \ominus \psi$
3. $\mathcal{K} \ominus (\phi \wedge \psi) = \mathcal{K} \ominus \phi \cap \mathcal{K} \ominus \psi$

Revision

The most studied form of belief change is that of revision. The idea is that after an agent has decided to accept some new information ϕ , he has to sensibly change his beliefs so that:

1. His beliefs remain consistent
2. He believes ϕ
3. He keeps believing as much as possible as he already did
4. He adopts as few as possible new beliefs

Revision Postulates

- ($\mathcal{K} \circledast 1$) $\mathcal{K} \circledast \phi$ is a belief set *type*
- ($\mathcal{K} \circledast 2$) $\phi \in \mathcal{K} \circledast \phi$ *success*
- ($\mathcal{K} \circledast 3$) $\mathcal{K} \circledast \phi \subseteq \mathcal{K} \oplus \phi$ *upper bound*
- ($\mathcal{K} \circledast 4$) if $\neg\phi \notin \mathcal{K}$ then $\mathcal{K} \oplus \phi \subseteq \mathcal{K} \circledast \phi$ *lower bound*
- ($\mathcal{K} \circledast 5$) $\mathcal{K} \circledast \phi = \mathcal{K}_\perp$ iff $\vdash \neg\phi$ *triviality*
- ($\mathcal{K} \circledast 6$) If $\vdash \phi \leftrightarrow \psi$ then $\mathcal{K} \circledast \phi = \mathcal{K} \circledast \psi$ *extensionality*
- ($\mathcal{K} \circledast 7$) $\mathcal{K} \circledast (\phi \wedge \psi) \subseteq (\mathcal{K} \circledast \phi) \oplus \psi$ *iterated 3*
- ($\mathcal{K} \circledast 8$) if $\neg\psi \notin \mathcal{K} \circledast \phi$ then $(\mathcal{K} \circledast \phi) \oplus \psi \subseteq \mathcal{K} \circledast (\phi \wedge \psi)$ *iterated 4*

Revision Postulates

Similar motivations to the expansion and contraction postulates.

The Levi Identity

Theorem 4

Suppose we have functions \oplus satisfying $(\mathcal{K} \oplus 1) - (\mathcal{K} \oplus 6)$ and \ominus satisfying $(\mathcal{K} \ominus 1) - (\mathcal{K} \ominus 8)$. If \otimes is defined as

$$\mathcal{K} \otimes \phi \equiv (\mathcal{K} \ominus \neg\phi) \oplus \phi$$

then it satisfies $(\mathcal{K} \otimes 1) - (\mathcal{K} \otimes 8)$.

Characterisation Results

So far we have not defined contraction (and revision) in a unique way. When contracting by a formula ϕ , we have many candidate belief sets as possible results. It makes sense to look for these among the sets that fail to imply ϕ but are otherwise maximal in \mathcal{K} .

Characterisation Results

So far we have not defined contraction (and revision) in a unique way. When contracting by a formula ϕ , we have many candidate belief sets as possible results. It makes sense to look for these among the sets that fail to imply ϕ but are otherwise maximal in \mathcal{K} .

The set of all maximal belief sets that fail to imply ϕ is denoted by $\mathcal{K} \perp \phi$.

Example

Suppose we only have 3 atoms p, q, r and $\mathcal{K} = Cn(p, q, r)$.

$$\mathcal{K} \perp (p \wedge q) = \{Cn(p, r), Cn(q, r), Cn(p, q \leftrightarrow r), \\ Cn(q, p \leftrightarrow r), Cn(p \leftrightarrow q, q \leftrightarrow r), Cn(p \leftrightarrow q, r)\}$$

Selection Function

From $\mathcal{K} \perp \phi$ we can find good candidates for $\mathcal{K} \ominus \phi$. If S is a function that selects a subset of this set, we can define contraction as below:

Definition 5

Let S be a selection function as described above. A partial meet contraction function \ominus_{pm} is defined as:

$$\mathcal{K} \ominus_{pm} \phi = \bigcap S(\mathcal{K} \perp \phi), \text{ if } \mathcal{K} \perp \phi \neq \emptyset$$

Partial Meet Contraction

Theorem 6

A contraction function \ominus satisfies the basic postulates $(\mathcal{K} \ominus 1) - (\mathcal{K} \ominus 6)$ iff it can be generated by a partial meet contraction.

Entrenchment

Entrenchment is a notion of how much "epistemic value" each formula has for the agent. Intuitively, if an agent is forced to give up either ϕ or ψ , he will give up the one he is less entrenched in. The notion of entrenchment will help us finally define contraction in a unique way that satisfies all the postulates.

Characterisation

Assume a binary relation \leq on the maximal sets of \mathcal{K} that fail to imply a formula (this means that the order does not depend on the formula we want to contract with).

A selection function is a marking-off identity for \leq if:

$$S(\mathcal{K} \perp \phi) = \{\mathcal{K}' \in (\mathcal{K} \perp \phi) \mid \mathcal{K}'' \leq \mathcal{K}' \forall \mathcal{K}'' \in (\mathcal{K} \perp \phi)\}$$

Characterisation

Assume a binary relation \leq on the maximal sets of \mathcal{K} that fail to imply a formula (this means that the order does not depend on the formula we want to contract with).

A selection function is a marking-off identity for \leq if:

$$S(\mathcal{K} \perp \phi) = \{\mathcal{K}' \in (\mathcal{K} \perp \phi) \mid \mathcal{K}'' \leq \mathcal{K}' \forall \mathcal{K}'' \in (\mathcal{K} \perp \phi)\}$$




Theorem 7

A contraction function is based on a selection function that is a marking-off identity of a transitive relation iff the contraction function satisfies $(\mathcal{K} \ominus 1) - (\mathcal{K} \ominus 8)$

References I

-  Carlos E. Alchourrón, Peter Gärdenfors, and David Makinson. “On the Logic of Theory Change: Partial Meet Contraction and Revision Functions”. In: *The Journal of Symbolic Logic* 50.2 (1985), pp. 510–530. ISSN: 00224812. URL: <http://www.jstor.org/stable/2274239>.
-  Alexandru Baltag and Bryan Renne. “Dynamic Epistemic Logic”. In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Winter 2016. Metaphysics Research Lab, Stanford University, 2016.
-  H. van Ditmarsch et al. *Handbook of Epistemic Logic*. College Publications, 2015. ISBN: 9781848901582. URL: <https://books.google.gr/books?id=tnj9rQEACAAJ>.
-  Hans van Ditmarsch, Wiebe van der Hoek, and Barteld Kooi. *Dynamic Epistemic Logic*. 1st. Springer Publishing Company, Incorporated, 2007. ISBN: 1402058381.

References II

-  Peter Gärdenfors. “Belief revision: An introduction”. In: *Belief Revision*. Ed. by Peter Gärdenfors. Cambridge Tracts in Theoretical Computer Science. Cambridge University Press, 1992, pp. 1–28. DOI: 10.1017/CB09780511526664.001.
-  Peter Gärdenfors. “Notes on the History of Ideas Behind AGM”. In: *Journal of Philosophical Logic* 40.2 (2011), pp. 115–120. ISSN: 00223611, 15730433. URL: <http://www.jstor.org/stable/41487509>.
-  Sven Ove Hansson. “Logic of Belief Revision”. In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Winter 2017. Metaphysics Research Lab, Stanford University, 2017.