Dynamic Epistemic Logic: Epistemic Actions

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Overview

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- Public announcements are 'updates' that convey the same information for all agents.
- In general, there are various more complex 'updates', or, as we call them, epistemic actions.
- They may convey different information for different agents.
- They may result even in the enlargement of the domain of the model (and its structure).

Motivating example: Buy or sell?

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- We model this by an epistemic state: Two states, one atom p for "United Agents is doing well".We assume that Anne (a) and Bill (b) are both uncertain about the value of p, and this is common knowledge.
- In fact, p is true.

The epistemic model for this we call *Letter*.

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- **(bothmayread)** Bill orders a drink at the bar while Anne goes to the bathroom. Each may have read the letter while the other was away from the table. (Both read the letter.) (United Agents is doing well.)

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Let's introduce a language in which we are able to express *all* the above actions...

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Syntax of $\mathcal{L}_!(A, P)$ (1/2)

To the language $\mathcal{L}_{KC}$ for multi-agent epistemic logic with common knowledge for a set A of agents and a set P of atomic propositions, we add dynamic modal operators for programs that are called epistemic actions or just actions.

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Formulas, actions, group (1/2)

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where

$p \in P$, $a \in A$, $B \subseteq A$,

$\psi \in \mathcal{L}_!^{stat}(gr(\alpha), P)$, $\beta \in \mathcal{L}_!^{act}(B, P)$, $\beta' \in \mathcal{L}_!^{act}(gr(\alpha), P)$.

Syntax of $\mathcal{L}_!(A, P)$ (2/2)

Formulas, actions, group (2/2)

The *group* $gr(\alpha)$ of an action α is defined as:

$gr(?\phi) = \emptyset$
$gr(L_B\alpha) = B$
$gr(\alpha!\alpha') = gr(\alpha)$
$gr(\alpha¡\alpha') = gr(\alpha')$
$gr(\alpha;\alpha') = gr(\alpha')$
$gr(\alpha \cup \alpha') = gr(\alpha) \cap gr(\alpha')$

Note: *group gr* keeps track of the agents occurring in learning operators in actions.

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Note: When we have more than one options in a local choice, we are able to write them as many local choices between two options. So there is no problem in **mayread** and in **bothmayread** above.

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Excercise: What are the types of the actions in the motivating example? (On board)

Equivalence of epistemic states

Definition

Let $M, M' \in \mathcal{S}5(\subseteq A)$, $s \in M$, $s' \in M'$ and $a \in A$. Then $(M, s) \sim_a (M', s')$ iff

- $a \notin gr(M) \cup gr(M')$ or
- $M = M'$ and $s \sim_a s'$ or
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Note: The epistemic states are the same from that agent's point of view.

Overview

Semantics (1/3)

Definition (1/2)

Let $M = \langle S, \sim, V \rangle \in \mathcal{S}5(A, P)$ and $s \in S$. The semantics of $\mathcal{L}_!^{stat}(A, P$ formulas and $\mathcal{L}_!^{act}(A, P)$ actions is defined as follows:

- $M, s \models p$ iff $s \in V_p$
- $M, s \models \neg\phi$ iff $M, s \not\models \phi$
- $M, s \models \phi \wedge \psi$ iff $M, s \models \phi$ and $M, s \models \psi$
- $M, s \models K_a\phi$ iff for all $s' \in S : s \sim_a s'$ implies $M, s' \models \phi$
- $M, s \models C_B\phi$ iff for all $s' \in S : s \sim_B s'$ implies $M, s' \models \phi$
- $M, s \models [\alpha]\phi$ iff for all $M', s' : (M, s)[\![\alpha]\!](M', s')$ implies $M', s' \models \phi$

Semantics (2/3)

Definition (2/2)

- $(M, s)[\![?\phi]\!](M', s')$ iff $M' = \langle [\![\phi]\!]_M, \emptyset, V|[\![\phi]\!]_M \rangle$ and $s = s'$
- $(M, s)[\![L_B\alpha]\!](M', s')$ iff $M' = \langle S', \sim', V' \rangle$ and $(M, s)[\![\alpha]\!](M', s')$
- $[\![\alpha; \alpha']\!] = [\![\alpha]\!] \circ [\![\alpha']\!]$
- $[\![\alpha \cup \alpha']\!] = [\![\alpha]\!] \cup [\![\alpha']\!]$
- $[\![\alpha ! \alpha']\!] = [\![\alpha]\!]$

Semantics (3/3): A few words of explanation

- In the clause for $[\alpha]_\phi$, $(M', s') \in \bullet\mathcal{S}5(\subseteq A, P)$.
- In the clause for $?\phi$, $(V|[\![\phi]\!]_M)_p = V_p \cap [\![\phi]\!]_M$.
- In the clause for $L_B\alpha$, $(M', s') \in \bullet\mathcal{S}5(B, P)$ such that $S' = \{(M'', s'') \mid \text{there is a } t \in S : (M, t)[\![\alpha_\cup]\!](M'', s'')\}$;

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  if $(M, s)[\![\alpha_\cup]\!](M''_1, s'')$ and $(M, t)[\![\alpha_\cup]\!](M''_2, t'')$, then for all $a \in B$

  $(M''_1, s'') \sim'_a (M''_2, t'')$ iff $s \sim_a t$ and $(M''_1, s'') \sim_a (M''_2, t'')$

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We call all the validities under this semantics the logic *EA*.

Exercise

Let's compute the interpretation of the action **read** for epistemic state $(Letter, 1)$. (*Note*: State 1, is the state where the value of p is $\top$.)

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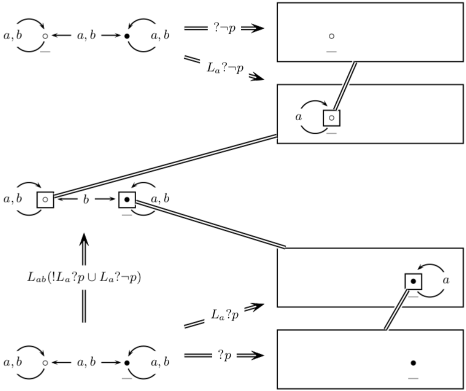

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Let's now see and discuss the result of execution of each action (**tell**, **read**, **mayread**, **bothmayread**) for epistemic state $(Letter, 1)$.

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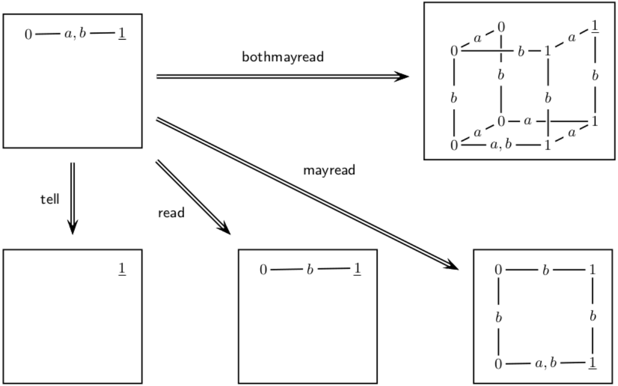

Examples (2/2)

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- Notice that after execution of **mayread** and of **bothmayread** in epistemic state $(Letter, 1)$ the resulting epistemic states are larger than the original.

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Theorem (Bisimilarity implies modal equivalence)

Let $\phi \in \mathcal{L}_!^{stat}(A)$. *Let* $(M, s), (M', s') \in \bullet\mathcal{S}5(A)$. *If* $(M, s) \leftrightarrow (M', s')$, *then* $(M, s) \models \phi$ *iff* $(M', s') \models \phi$.

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Theorem (Action execution preserves bisimilarity)

Let $\alpha \in \mathcal{L}_!^{act}(A)$ and $(M, s), (M', s') \in \bullet\mathcal{S}5(A)$. If $(M, s) \Leftrightarrow (M', s')$ and there is a $(N, t) \in \bullet\mathcal{S}5(\subseteq A)$ such that $(M, s)[\![\alpha]\!](N, t)$ then there is a $(N', t') \in \bullet\mathcal{S}5(\subseteq A)$ such that $(M', s')[\![\alpha]\!](N', t')$ and $(N, t) \Leftrightarrow (N', t')$.

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Let $\phi \in \mathcal{L}_!^{stat}(A)$*. Let* $(M, s), (M', s') \in \bullet\mathcal{S}5(A)$*. If* $(M, s) \leftrightarrows (M', s')$*, then* $(M, s) \models \phi$ *iff* $(M', s') \models \phi$*.*

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(*Proof: By induction on the ("complexity" of the) structure of* α*.*)

Notes

- The logic *EA* is useful in modeling card games and spreading gossip.
- The result of execution of an action could be a much more "complex" epistemic state than the original.
- Apparently, it is possible to execute an action for the resulting epistemic state of an execution of an action and so on.

Overview

References

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