Dynamic Epistemic Logic: Action Models

Thomas Pipilikas

INTER-INSTITUTIONAL GRADUATE PROGRAM “ALGORITHMS, LOGIC AND DISCRETE MATHEMATICS”

Overview

Bisimilarity in Kripke Models

Definition (Bisimulation)

Let two Kripke models $M = \langle S, R, V \rangle$ and $M' = \langle S', R', V' \rangle$ be given. A non-empty relation $\Re \subseteq S \times S'$ is a *bisimulation* iff for all $s \in S$ and $s' \in S'$ with $(s, s') \in \Re$:

- **atoms** $s \in V(p)$ iff $s' \in V'(p)$, for any $p \in P$
- **forth** for all $a \in A$ and all $t \in S$, if $(s, t) \in R_a$, then there is a $t' \in S'$ such that $(s', t') \in R'_a$ and $(t, t') \in \Re$
- **back** for all $a \in A$ and all $t' \in S'$, if $(s', t') \in R'_a$, then there is a $t \in S$ such that $(s, t) \in R_a$ and $(t, t') \in \Re$

We write $(M, s) \leftrightarrows (M', s')$, iff there is a bisimulation between M and M' linking s and s'. Then we call (M, s) and (M', s') bisimilar.

Bisimilarity in Kripke Models

Definition (Bisimulation)

Let two Kripke models $M = \langle S, R, V \rangle$ and $M' = \langle S', R', V' \rangle$ be given. A non-empty relation $\mathfrak{R} \subseteq S \times S'$ is a *bisimulation* iff for all $s \in S$ and $s' \in S'$ with $(s, s') \in \mathfrak{R}$:

atoms $s \in V(p)$ iff $s' \in V'(p)$, for any $p \in P$

forth for all $a \in A$ and all $t \in S$, if $(s, t) \in R_a$, then there is a $t' \in S'$ such that $(s', t') \in R'_a$ and $(t, t') \in \mathfrak{R}$

back for all $a \in A$ and all $t' \in S'$, if $(s', t') \in R'_a$, then there is a $t \in S$ such that $(s, t) \in R_a$ and $(t, t') \in \mathfrak{R}$

We write $(M, s) \leftrightarroweq (M', s')$, iff there is a bisimulation between M and M' linking s and s'. Then we call (M, s) and (M', s') bisimilar.

Theorem (2.15)

For all pointed models (M, s) *and* (M', s'), *if* $(M, s) \leftrightarroweq (M', s')$, *then* $(M, s) \equiv_{\mathcal{L}_K} (M', s')$; *i.e. for any* $\varphi \in \mathcal{L}_K$ $M, s \models \varphi$ *iff* $M', s' \models \varphi$.

Preservation of bisimilarity

If we execute the same action in two given bisimilar epistemic states, we would want them to be bisimilar again.

Preservation of bisimilarity

If we execute the same action in two given bisimilar epistemic states, we would want them to be bisimilar again.

This is indeed the case!

Preservation of bisimilarity

If we execute the same action in two given bisimilar epistemic states, we would want them to be bisimilar again.

This is indeed the case!

Proposition (6.21)

Given epistemic states (M, s) *and* (M', s') *s.t.* $(M, s) \underline{\leftrightarrow} (M', s')$*. Let* (M, s) *with* $\mathsf{M} = \langle \mathsf{S}, \sim, \mathsf{pre} \rangle$ *be executable in* (M, s)*. Then*

$$(M \otimes \mathsf{M}, (s, \mathsf{s})) \underline{\leftrightarrow} (M' \otimes \mathsf{M}, (s', \mathsf{s}))$$

Proof of Proposition 6.21 (1/2)

(M, s) executable in $(M, s) \iff$

Proof of Proposition 6.21 (1/2)

(M, s) executable in $(M, s) \iff$

$M, s \models \mathsf{pre}(\mathsf{s}) \overset{\text{Th.2.15}}{\iff}$

Proof of Proposition 6.21 (1/2)

(M, s) executable in $(M, s) \iff$

$M, s \models \mathsf{pre}(\mathsf{s}) \overset{\text{Th.2.15}}{\iff}$

$M', s' \models \mathsf{pre}(\mathsf{s}) \iff$

Proof of Proposition 6.21 (1/2)

(M, s) executable in $(M, s) \iff$

$M, s \models \mathsf{pre}(\mathsf{s}) \overset{\text{Th.2.15}}{\iff}$

$M', s' \models \mathsf{pre}(\mathsf{s}) \iff$

(M, s) executable in (M', s')

Proof of Proposition 6.21 (1/2)

(M, s) executable in (M, s) $\iff$

$M, s \models \mathsf{pre}(\mathsf{s}) \overset{\text{Th.2.15}}{\iff}$

$M', s' \models \mathsf{pre}(\mathsf{s}) \iff$

(M, s) executable in (M', s')

Let $\mathfrak{R} : (M, s) \leftrightarroweq (M', s')$ bisimulation.

Proof of Proposition 6.21 (1/2)

(M, s) executable in $(M, s) \iff$

$M, s \models \mathsf{pre}(\mathsf{s}) \overset{\text{Th.2.15}}{\iff}$

$M', s' \models \mathsf{pre}(\mathsf{s}) \iff$

(M, s) executable in (M', s')

Let $\mathfrak{R} : (M, s) \underline{\leftrightarrow} (M', s')$ bisimulation.

We define the relation

$$\mathfrak{R}' := \left\{((t, \mathsf{t}), (t', \mathsf{t}')) \in S_{\otimes \mathsf{M}} \times S'_{\otimes \mathsf{M}} \mid t \,\mathfrak{R}\, t' \;\&\; \mathsf{t} = \mathsf{t}'\right\}$$

i.e.

$$(t, \mathsf{t})\,\mathfrak{R}'\,(t', \mathsf{t}') \iff t \,\mathfrak{R}\, t' \;\&\; \mathsf{t} = \mathsf{t}'$$

Proof of Proposition 6.21 (2/2)

Let $(s, \mathsf{s})\, \mathfrak{R}'\, (s', \mathsf{s}')$

Proof of Proposition 6.21 (2/2)

Let $(s, \mathsf{s})\, \mathfrak{R}'\, (s', \mathsf{s}')$

atoms Let $(s, \mathsf{s})\, \mathfrak{R}'\, (s', \mathsf{s}')$

$(s, \mathsf{s}) \in V_{\otimes \mathsf{M}}(p)$

Proof of Proposition 6.21 (2/2)

Let $(s, \mathsf{s}) \, \mathfrak{R}' \, (s', \mathsf{s}')$

atoms Let $(s, \mathsf{s}) \, \mathfrak{R}' \, (s', \mathsf{s}')$

$$(s, \mathsf{s}) \in V_{\otimes \mathsf{M}}(p) \iff s \in V(p)$$

Proof of Proposition 6.21 (2/2)

Let $(s, \mathsf{s}) \, \mathfrak{R}' \, (s', \mathsf{s}')$

atoms Let $(s, \mathsf{s}) \, \mathfrak{R}' \, (s', \mathsf{s}')$

$(s, \mathsf{s}) \in V_{\otimes \mathsf{M}}(p) \iff s \in V(p) \overset{s \, \mathfrak{R} \, s'}{\iff} s' \in V'(p)$

Proof of Proposition 6.21 (2/2)

Let $(s, \mathsf{s})\,\Re'\,(s', \mathsf{s}')$

atoms Let $(s, \mathsf{s})\,\Re'\,(s', \mathsf{s}')$

$$(s, \mathsf{s}) \in V_{\otimes\mathsf{M}}(p) \iff s \in V(p) \overset{s\,\Re\,s'}{\iff} s' \in V'(p) \iff (s', \mathsf{s}) \in V'_{\otimes\mathsf{M}}(p)$$

Proof of Proposition 6.21 (2/2)

Let $(s, \mathsf{s}) \, \mathfrak{R}' \, (s', \mathsf{s}')$

atoms Let $(s, \mathsf{s}) \, \mathfrak{R}' \, (s', \mathsf{s}')$

$$(s, \mathsf{s}) \in V_{\otimes \mathrm{M}}(p) \iff s \in V(p) \overset{s \, \mathfrak{R} \, s'}{\iff} s' \in V'(p) \iff (s', \mathsf{s}) \in V'_{\otimes \mathrm{M}}(p)$$

forth Let agent $a \in A$ and state (t, t) s.t. $(t, \mathsf{t}) \sim_a^{\otimes \mathrm{M}} (s, \mathsf{s})$

Proof of Proposition 6.21 (2/2)

Let $(s, \mathsf{s}) \, \Re' \, (s', \mathsf{s}')$

atoms Let $(s, \mathsf{s}) \, \Re' \, (s', \mathsf{s}')$

$$(s, \mathsf{s}) \in V_{\otimes \mathsf{M}}(p) \iff s \in V(p) \overset{s \, \Re \, s'}{\iff} s' \in V'(p) \iff (s', \mathsf{s}) \in V'_{\otimes \mathsf{M}}(p)$$

forth Let agent $a \in A$ and state (t, t) s.t. $(t, \mathsf{t}) \sim_a^{\otimes \mathsf{M}} (s, \mathsf{s})$; i.e. $t \sim_a s$ and $\mathsf{t} \sim_a \mathsf{s}$.

Proof of Proposition 6.21 (2/2)

Let $(s, \mathsf{s}) \, \mathfrak{R}' \, (s', \mathsf{s}')$

atoms Let $(s, \mathsf{s}) \, \mathfrak{R}' \, (s', \mathsf{s}')$

$$(s, \mathsf{s}) \in V_{\otimes \mathsf{M}}(p) \iff s \in V(p) \overset{s \, \mathfrak{R} \, s'}{\iff} s' \in V'(p) \iff (s', \mathsf{s}) \in V'_{\otimes \mathsf{M}}(p)$$

forth Let agent $a \in A$ and state (t, t) s.t. $(t, \mathsf{t}) \sim_a^{\otimes \mathsf{M}} (s, \mathsf{s})$; i.e. $t \sim_a s$ and $\mathsf{t} \sim_a \mathsf{s}$.

By $\mathfrak{R} : (M, s) \leftrightarrows (M', s')$ there is $t' \in S'$ s.t. $t \, \mathfrak{R} \, t'$ and $t' \sim'_a s'$.

Proof of Proposition 6.21 (2/2)

Let $(s, \mathsf{s}) \,\mathfrak{R}'\, (s', \mathsf{s}')$

atoms Let $(s, \mathsf{s}) \,\mathfrak{R}'\, (s', \mathsf{s}')$

$(s, \mathsf{s}) \in V_{\otimes \mathsf{M}}(p) \iff s \in V(p) \stackrel{s \,\mathfrak{R}\, s'}{\iff}$
$s' \in V'(p) \iff (s', \mathsf{s}) \in V'_{\otimes \mathsf{M}}(p)$

forth Let agent $a \in A$ and state (t, t) s.t. $(t, \mathsf{t}) \sim_a^{\otimes \mathsf{M}} (s, \mathsf{s})$; i.e. $t \sim_a s$ and $\mathsf{t} \sim_a \mathsf{s}$.

By $\mathfrak{R}$: $(M, s) \,\underline{\leftrightarrow}\, (M', s')$ there is $t' \in S'$ s.t. $t \,\mathfrak{R}\, t'$ and $t' \sim_a' s'$.

Trivially, $(t, \mathsf{t}) \,\mathfrak{R}'\, (t', \mathsf{t})$ and $(t', \mathsf{t}) \sim_a'^{\otimes \mathsf{M}} (s', \mathsf{s}')$

Proof of Proposition 6.21 (2/2)

Let $(s, \mathsf{s})\,\mathfrak{R}'\,(s', \mathsf{s}')$

atoms Let $(s, \mathsf{s})\,\mathfrak{R}'\,(s', \mathsf{s}')$

$(s, \mathsf{s}) \in V_{\otimes \mathsf{M}}(p) \iff s \in V(p) \overset{s\,\mathfrak{R}\,s'}{\iff} s' \in V'(p) \iff (s', \mathsf{s}) \in V'_{\otimes \mathsf{M}}(p)$

forth Let agent $a \in A$ and state (t, t) s.t. $(t, \mathsf{t}) \sim_a^{\otimes \mathsf{M}} (s, \mathsf{s})$; i.e. $t \sim_a s$ and $\mathsf{t} \sim_a \mathsf{s}$.

By $\mathfrak{R}$: $(M, s) \leftrightarroweq (M', s')$ there is $t' \in S'$ s.t. $t\,\mathfrak{R}\,t'$ and $t' \sim'_a s'$.

Trivially, $(t, \mathsf{t})\,\mathfrak{R}'\,(t', \mathsf{t})$ and $(t', \mathsf{t}) \sim_a'^{\otimes \mathsf{M}} (s', \mathsf{s}')$

back Similarly, with **forth**

$\square$

Bisimulation of Actions

As the action models also have structure, we can go beyond such observations.

Bisimulation of Actions

As the action models also have structure, we can go beyond such observations.
The obvious notion of bisimilarity for action models is as for epistemic states, but with the requirement that points have corresponding *valuations* replaced by the requirement that points have corresponding *preconditions*.

Bisimulation of Actions

As the action models also have structure, we can go beyond such observations.
The obvious notion of bisimilarity for action models is as for epistemic states, but with the requirement that points have corresponding *valuations* replaced by the requirement that points have corresponding *preconditions*.

Definition (Bisimulation of Actions)

Given are pointed action models (M, u) with $\mathsf{M} = \langle \mathsf{S}, \sim, \mathsf{pre} \rangle$, and $(\mathsf{M}', \mathsf{u}')$ with $\mathsf{M}' = \langle \mathsf{S}', \sim', \mathsf{pre}' \rangle$. A bisimulation between (M, u) and $(\mathsf{M}', \mathsf{u}')$ is a relation $\Re \subseteq \mathsf{S} \times \mathsf{S}'$ s.t. $\mathsf{u} \, \Re \, \mathsf{u}'$ and s.t. the following three conditions are met for each agent a (for arbitrary action points):

Forth If $\mathsf{s} \, \Re \, \mathsf{s}'$ and $\mathsf{s} \sim_a \mathsf{t}$, then there is an $\mathsf{t}' \in \mathsf{S}'$ s.t. $\mathsf{t} \, \Re \, \mathsf{t}'$ and $\mathsf{s}' \sim'_a \mathsf{t}'$.

Back If $\mathsf{s} \, \Re \, \mathsf{s}'$ and $\mathsf{s}' \sim'_a \mathsf{t}'$, then there is an $\mathsf{t} \in \mathsf{S}$ s.t. $\mathsf{t} \, \Re \, \mathsf{t}'$ and $\mathsf{s} \sim_a \mathsf{t}$.

Pre If $\mathsf{s} \Re \mathsf{s}'$, then $\models \mathsf{pre}(\mathsf{s}) \leftrightarrow \mathsf{pre}'(\mathsf{s}')$

Preservation of Bisimilarity for Actions

- A relation $\mathfrak{R}$ is a *total bisimulation* between M and M' iff for each $\mathsf{s} \in \mathsf{S}$ there is an $\mathsf{s}' \in \mathsf{S}'$ s.t. $\mathfrak{R}$ is a bisimulation between (M, s) and $(\mathsf{M}', \mathsf{s}')$, and vice versa.

Preservation of Bisimilarity for Actions

- A relation $\mathfrak{R}$ is a *total bisimulation* between M and M' iff for each $\mathsf{s} \in \mathsf{S}$ there is an $\mathsf{s}' \in \mathsf{S}'$ s.t. $\mathfrak{R}$ is a bisimulation between (M, s) and $(\mathsf{M}', \mathsf{s}')$, and vice versa.
- As usual we write $(\mathsf{M}, \mathsf{s}) \underline{\leftrightarrow} (\mathsf{M}', \mathsf{s}')$ if such a bisimulation exists; or $\mathfrak{R} : (\mathsf{M}, \mathsf{s}) \underline{\leftrightarrow} (\mathsf{M}', \mathsf{s}')$, to make the bisimulation explicit.

Preservation of Bisimilarity for Actions

- A relation $\mathfrak{R}$ is a *total bisimulation* between M and M' iff for each $\mathsf{s} \in \mathsf{S}$ there is an $\mathsf{s}' \in \mathsf{S}'$ s.t. $\mathfrak{R}$ is a bisimulation between (M, s) and $(\mathsf{M}', \mathsf{s}')$, and vice versa.
- As usual we write $(\mathsf{M}, \mathsf{s}) \underline{\leftrightarrow} (\mathsf{M}', \mathsf{s}')$ if such a bisimulation exists; or $\mathfrak{R} : (\mathsf{M}, \mathsf{s}) \underline{\leftrightarrow} (\mathsf{M}', \mathsf{s}')$, to make the bisimulation explicit.

If two bisimilar action models are executed in the same epistemic state, are the resulting epistemic states bisimilar?

Preservation of Bisimilarity for Actions

- A relation $\mathfrak{R}$ is a *total bisimulation* between M and M' iff for each $\mathsf{s} \in \mathsf{S}$ there is an $\mathsf{s}' \in \mathsf{S}'$ s.t. $\mathfrak{R}$ is a bisimulation between (M, s) and $(\mathsf{M}', \mathsf{s}')$, and vice versa.
- As usual we write $(\mathsf{M}, \mathsf{s}) \leftrightarroweq (\mathsf{M}', \mathsf{s}')$ if such a bisimulation exists; or $\mathfrak{R} : (\mathsf{M}, \mathsf{s}) \leftrightarroweq (\mathsf{M}', \mathsf{s}')$, to make the bisimulation explicit.

If two bisimilar action models are executed in the same epistemic state, are the resulting epistemic states bisimilar?

Proposition (6.23)

Given two action models s.t. $(\mathsf{M}, \mathsf{s}) \leftrightarroweq (\mathsf{M}', \mathsf{s}')$ *and an epistemic state* (M, s), *s.t.* (M, s) *is executable in* (M, s). *Then*

$$(M \otimes \mathsf{M}, (s, \mathsf{s})) \leftrightarroweq (M \otimes \mathsf{M}', (s, \mathsf{s}'))$$

Preservation of Bisimilarity for Actions

- A relation $\mathfrak{R}$ is a *total bisimulation* between M and M' iff for each $\mathsf{s} \in \mathsf{S}$ there is an $\mathsf{s}' \in \mathsf{S}'$ s.t. $\mathfrak{R}$ is a bisimulation between (M, s) and $(\mathsf{M}', \mathsf{s}')$, and vice versa.
- As usual we write $(\mathsf{M}, \mathsf{s}) \leftrightarroweq (\mathsf{M}', \mathsf{s}')$ if such a bisimulation exists; or $\mathfrak{R} : (\mathsf{M}, \mathsf{s}) \leftrightarroweq (\mathsf{M}', \mathsf{s}')$, to make the bisimulation explicit.

If two bisimilar action models are executed in the same epistemic state, are the resulting epistemic states bisimilar?

Proposition (6.23)

Given two action models s.t. $(\mathsf{M}, \mathsf{s}) \leftrightarroweq (\mathsf{M}', \mathsf{s}')$ *and an epistemic state* (M, s), *s.t.* (M, s) *is executable in* (M, s). *Then*

$$(M \otimes \mathsf{M}, (s, \mathsf{s})) \leftrightarroweq (M \otimes \mathsf{M}', (s, \mathsf{s}'))$$

Hint: $\mathfrak{R}' := \{((t, \mathsf{t}), (t', \mathsf{t}')) \in S_{\otimes \mathsf{M}} \times S_{\otimes \mathsf{M}'} \mid t = t' \ \& \ \mathsf{t} \, \mathfrak{R} \, \mathsf{t}'\}$

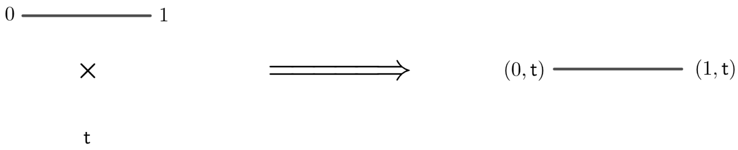
It turns out, however, that this requirement for action sameness is too strong: if we merely want to guarantee that the resulting epistemic states are bisimilar given two executed actions, then a weaker notion of sameness is already sufficient.

It turns out, however, that this requirement for action sameness is too strong: if we merely want to guarantee that the resulting epistemic states are bisimilar given two executed actions, then a weaker notion of sameness is already sufficient.

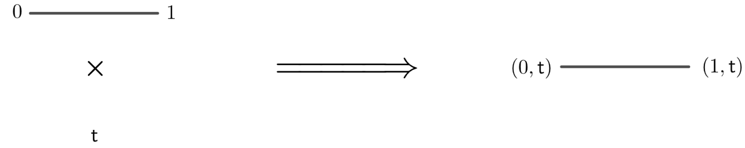
For example, consider

- the action model $\langle \{\mathsf{t}\}, \sim, \mathsf{pre} \rangle$ that is reflexive for all agents and with $\mathsf{pre}(\mathsf{t}) = \top$
- the action model $\langle \{\mathsf{np}, \mathsf{p}\}, \sim', \mathsf{pre}' \rangle$ such that no agent can distinguish between p and np, and with $\mathsf{pre}(\mathsf{p}) = p$ and $\mathsf{pre}(\mathsf{np}) = \neg p$

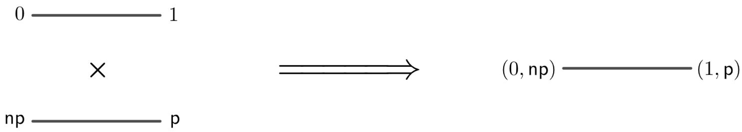
$$\langle \{\mathsf{t}\}, \sim, \mathsf{pre} \rangle$$

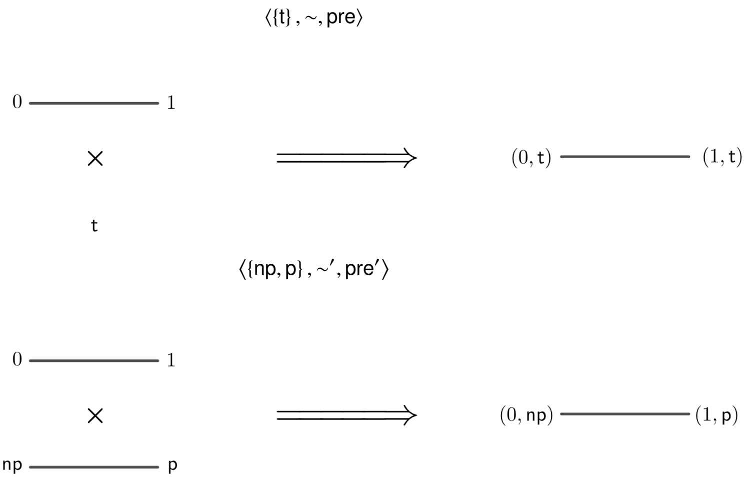


$\langle \{\mathsf{t}\}, \sim, \mathsf{pre} \rangle$



$\langle \{\mathsf{np}, \mathsf{p}\}, \sim', \mathsf{pre}' \rangle$





The final models are bisimilar (equivalent), but the action models weren't!

Action Emulation

Definition

Given are pointed action models (M, u) with $\mathsf{M} = \langle \mathsf{S}, \sim, \mathsf{pre} \rangle$, and $(\mathsf{M}', \mathsf{u}')$ with $\mathsf{M}' = \langle \mathsf{S}', \sim', \mathsf{pre}' \rangle$. A *emulation* between (M, u) and $(\mathsf{M}', \mathsf{u}')$ is a relation $\mathfrak{E} \subseteq \mathsf{S} \times \mathsf{S}'$ s.t. $\mathsf{u} \mathfrak{E} \mathsf{u}'$ and s.t. the following three conditions are met for each agent a (for arbitrary action points):

Forth If $\mathsf{s} \mathfrak{E} \mathsf{s}'$ and $\mathsf{s} \sim_a \mathsf{t}$, then there are $\mathsf{t}'_1, \ldots, \mathsf{t}'_n \in \mathsf{S}'$ s.t. for all $i \in [n]$, $\mathsf{t} \mathfrak{E} \mathsf{t}'_i$ and $\mathsf{s}' \sim'_a \mathsf{t}'_i$ and s.t. $\mathsf{pre}(\mathsf{t}) \models \mathsf{pre}'(\mathsf{t}'_1) \vee \cdots \vee \mathsf{pre}'(\mathsf{t}'_n)$.

Back If $\mathsf{s} \mathfrak{E} \mathsf{s}'$ and $\mathsf{s}' \sim'_a \mathsf{t}'$ then there are $\mathsf{t}_1, \ldots, \mathsf{t}_n \in \mathsf{S}$ s.t. for all $i \in [n]$, $\mathsf{t}_i \mathfrak{E} \mathsf{t}'$ and $\mathsf{s} \sim_a \mathsf{t}_i$ and s.t. $\mathsf{pre}'(\mathsf{t}') \models \mathsf{pre}(\mathsf{t}_1) \vee \cdots \vee \mathsf{pre}(\mathsf{t}_n)$.

Pre If $\mathsf{s} \mathfrak{E} \mathsf{s}'$, then $\mathsf{pre}(\mathsf{s}) \wedge \mathsf{pre}'(\mathsf{s}')$ is consistent.

A *total emulation* $\mathfrak{E} : \mathsf{M} \rightleftarrows \mathsf{M}'$ is an emulation such that for each $\mathsf{s} \in \mathsf{S}$ there is a $\mathsf{s}' \in \mathsf{S}'$, with $\mathsf{s} \mathfrak{E} \mathsf{s}'$ and vice versa.

Action Emulation

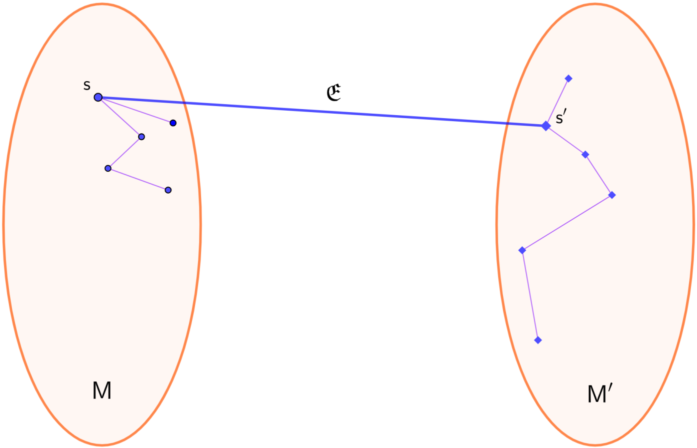
- In the previous definition, it is essential that the accessibility relations are *reflexive* (as they are equivalence relations).
- This ensures that the entailment requirements in the forth and back conditions also hold in the designated points of the structures

Bisimulation vs Action Emulation

We can paraphrase the difference between action bisimulation and action emulation as follows:

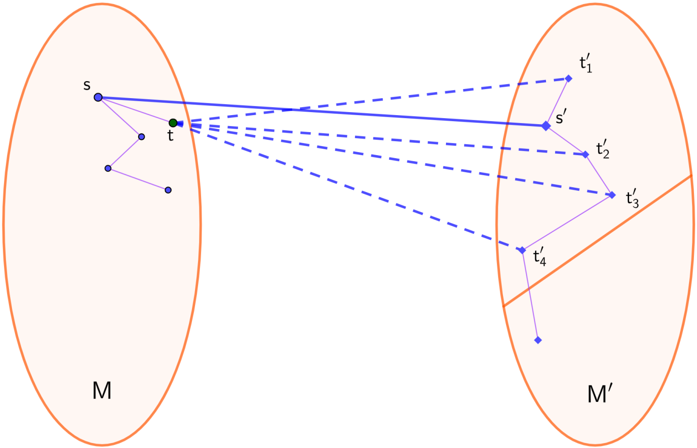
- Two bisimilar actions s, s' must have logically *equivalent* preconditions; i.e. $\models \mathsf{pre}(\mathsf{s}) \leftrightarrow \mathsf{pre}'(\mathsf{s}')$.
- In the case of two emulous actions it may be that one precondition only entails the other; i.e. $\models \mathsf{pre}(\mathsf{s}) \rightarrow \mathsf{pre}'(\mathsf{s}')$ but $\not\models \mathsf{pre}'(\mathsf{s}') \rightarrow \mathsf{pre}(\mathsf{s})$.
  In that case, formula $\mathsf{pre}'(\mathsf{s}')$ is strictly weaker than $\mathsf{pre}(\mathsf{s})$. This does not hurt if we can make up for the difference by finding sufficient emulous 'alternatives' $\mathsf{t}_1, \ldots, \mathsf{t}_n$ (including s) to s s.t. even though $\not\models \mathsf{pre}'(\mathsf{s}') \rightarrow \mathsf{pre}(\mathsf{s})$, after all $\models \mathsf{pre}'(\mathsf{s}') \rightarrow \mathsf{pre}(\mathsf{t}_1) \vee \cdots \vee \mathsf{pre}(\mathsf{t}_n)$

Action Emulation



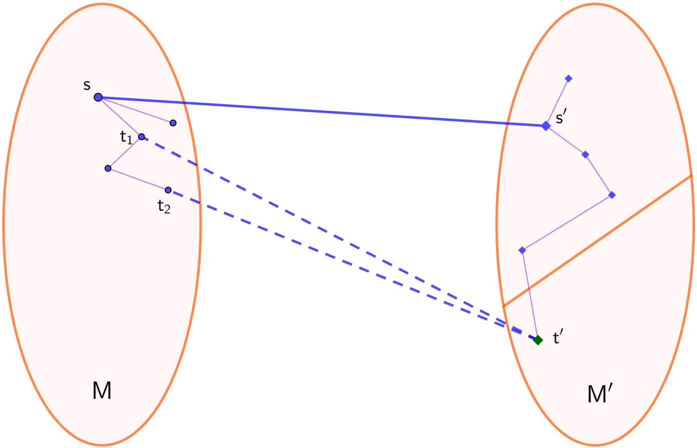
Action Emulation

Forth



Action Emulation

Back



An Alternative Emulation

Definition (Action Emulation 2)

Given are action models (M,u) with $\mathsf{M} = \langle \mathsf{S}, \sim, \mathsf{pre}\rangle$, and $(\mathsf{M}',\mathsf{u}')$ with $\mathsf{M}' = \langle \mathsf{S}', \sim', \mathsf{pre}'\rangle$. An emulation between (M,u) and $(\mathsf{M}',\mathsf{u}')$ is a relation $\mathfrak{E} \subseteq \mathsf{S} \times \mathsf{S}'$ s.t. $\mathsf{u}\ \mathfrak{E}\ \mathsf{u}'$ and s.t. the following three conditions are met for each agent a (for arbitrary action points):

Forth If $\mathsf{s}\ \mathfrak{E}\ \mathsf{s}'$ and $\mathsf{s} \sim_a \mathsf{t}$, then there is an $\mathsf{t}' \in \mathsf{S}'$ s.t. $\mathsf{t}\ \mathfrak{E}\ \mathsf{t}'$ and $\mathsf{s}' \sim'_a \mathsf{t}'$.

Back If $\mathsf{s}\ \mathfrak{E}\ \mathsf{s}'$ and $\mathsf{s}' \sim'_a \mathsf{t}'$, then there is an $\mathsf{t} \in \mathsf{S}$ s.t. $\mathsf{t}\ \mathfrak{E}\ \mathsf{t}'$ and $\mathsf{s} \sim_a \mathsf{t}$.

Pre If $\mathsf{s}\ \mathfrak{E}\ \mathsf{s}'$, then there are $\mathsf{s}'_1, \dots, \mathsf{s}'_n \in \mathsf{S}'$ including s' s.t. for all $i \in [n]$ $\mathsf{s}\ \mathfrak{E}\ \mathsf{s}'_i$ and $\mathsf{pre}(\mathsf{s}) \models \mathsf{pre}'\big(\mathsf{s}'_1\big) \vee \cdots \vee \mathsf{pre}'(\mathsf{s}'_n)$; and there are $\mathsf{s}_1, \dots, \mathsf{s}_n \in \mathsf{S}$ including S s.t. for all $i \in [n]$ $\mathsf{s}_i\ \mathfrak{E}\ \mathsf{s}'$ and $\mathsf{pre}'(\mathsf{s}') \models \mathsf{pre}(\mathsf{s}_1) \vee \cdots \vee \mathsf{pre}(\mathsf{s}_n)$

An Alternative Emulation

It is easy (EZ) to observe that the first definition of emulation, implies the second.

An Alternative Emulation

It is easy (EZ) to observe that the first definition of emulation, implies the second.
Are the two definitions equivalent?

An Alternative Emulation

It is easy (EZ) to observe that the first definition of emulation, implies the second.
Are the two definitions equivalent?
...

An Alternative Emulation

It is easy (EZ) to observe that the first definition of emulation, implies the second.

Are the two definitions equivalent?

...

Nope! Crash gonna crash them all!

Example 6.26

Consider the previous example $\mathcal{S}5$ action models:

- $\langle \{\mathsf{t}\}, \sim, \mathsf{pre} \rangle$, with $\mathsf{pre}(\mathsf{t}) = \top$
- $\langle \{\mathsf{np}, \mathsf{p}\}, \sim', \mathsf{pre}' \rangle$, with $\mathsf{pre}(\mathsf{p}) = p$ and $\mathsf{pre}(\mathsf{np}) = \neg p$.

Example 6.26

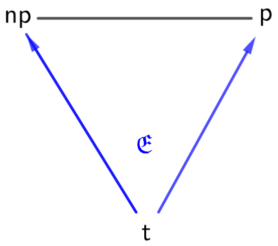
Consider the previous example $\mathcal{S}5$ action models:

- $\langle \{\mathsf{t}\}, \sim, \mathsf{pre} \rangle$, with $\mathsf{pre}(\mathsf{t}) = \top$
- $\langle \{\mathsf{np}, \mathsf{p}\}, \sim', \mathsf{pre}' \rangle$, with $\mathsf{pre}(\mathsf{p}) = p$ and $\mathsf{pre}(\mathsf{np}) = \neg p$.

It is easy to observe that the relation

$$\mathfrak{E} := \{(\mathsf{t}, \mathsf{np}), (\mathsf{t}, \mathsf{p})\}$$

is an emulation.



Example 6.26

Forth $\mathsf{pre(t)} \models \mathsf{pre'(np)} \vee \mathsf{pre'(p)}$

Example 6.26

Forth $\mathsf{pre(t)} \models \mathsf{pre'(np)} \vee \mathsf{pre'(p)} \equiv \neg p \vee p$

Example 6.26

Forth $\text{pre}(\text{t}) \models \text{pre}'(\text{np}) \vee \text{pre}'(\text{p}) \equiv \neg p \vee p \equiv \top$

Back $\varphi \models \text{pre}(\text{t})$

Example 6.26

Forth $\mathsf{pre}(\mathsf{t}) \models \mathsf{pre}'(\mathsf{np}) \vee \mathsf{pre}'(\mathsf{p}) \equiv \neg p \vee p \equiv \top$

Back $\varphi \models \mathsf{pre}(\mathsf{t}) \equiv \top$

Pre $\mathsf{pre}(\mathsf{t}) \wedge \mathsf{pre}'(\mathsf{np})$

Example 6.26

Forth $\mathsf{pre}(\mathsf{t}) \models \mathsf{pre}'(\mathsf{np}) \vee \mathsf{pre}'(\mathsf{p}) \;\equiv \neg p \vee p \equiv \top$

Back $\varphi \models \mathsf{pre}(\mathsf{t}) \;\equiv \top$

Pre $\mathsf{pre}(\mathsf{t}) \wedge \mathsf{pre}'(\mathsf{np}) \;\equiv \top \wedge \neg p$

Example 6.26

Forth $\mathsf{pre}(\mathsf{t}) \models \mathsf{pre}'(\mathsf{np}) \vee \mathsf{pre}'(\mathsf{p}) \equiv \neg p \vee p \equiv \top$

Back $\varphi \models \mathsf{pre}(\mathsf{t}) \equiv \top$

Pre $\mathsf{pre}(\mathsf{t}) \wedge \mathsf{pre}'(\mathsf{np}) \equiv \top \wedge \neg p \equiv \neg p$ consistent

Example 6.26

Forth $\mathsf{pre}(\mathsf{t}) \models \mathsf{pre}'(\mathsf{np}) \vee \mathsf{pre}'(\mathsf{p}) \equiv \neg p \vee p \equiv \top$

Back $\varphi \models \mathsf{pre}(\mathsf{t}) \equiv \top$

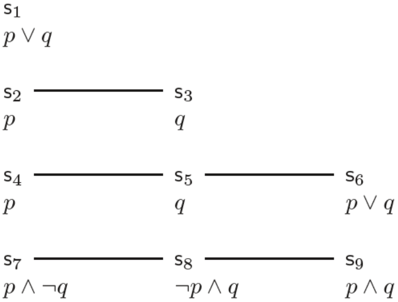
Pre $\mathsf{pre}(\mathsf{t}) \wedge \mathsf{pre}'(\mathsf{np}) \equiv \top \wedge \neg p \equiv \neg p$ consistent

$\mathsf{pre}(\mathsf{t}) \wedge \mathsf{pre}'(\mathsf{p}) \equiv p$ consistent

$\square$

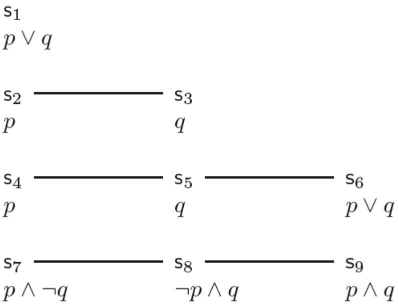
Exercise 6.27

Show that the following four action models are emulous. The preconditions of action points are indicated below their names.



Exercise 6.27

Show that the following four action models are emulous. The preconditions of action points are indicated below their names.



$$p \vee q \equiv \mathrm{pre}(s_1) \equiv \mathrm{pre}(s_2) \vee \mathrm{pre}(s_3) \equiv \mathrm{pre}(s_4) \vee \mathrm{pre}(s_5) \vee \mathrm{pre}(s_6)$$
$$\equiv \mathrm{pre}(s_7) \vee \mathrm{pre}(s_8) \vee \mathrm{pre}(s_9)$$

Emulation Guarantees Bisimilarity

Proposition (6.29 | Bisimilar actions are emulous)

A bisimulation $\mathfrak{R} : (\mathsf{M}, \mathsf{s}) \underline{\leftrightarrow} (\mathsf{M}', \mathsf{s}')$ *is also an emulation.*

Emulation Guarantees Bisimilarity

Proposition (6.29 | Bisimilar actions are emulous)

A bisimulation $\mathfrak{R} : (\mathsf{M}, \mathsf{s}) \leftrightarroweq (\mathsf{M}', \mathsf{s}')$ *is also an emulation.*

Proposition (6.30 | Emulation guarantees bisimilarity)

Given an epistemic model M *and action models* $\mathsf{M} \rightleftarrows \mathsf{M}'$. *Then*

$$\mathsf{M} \rightleftarrows \mathsf{M}' \Rightarrow M \otimes \mathsf{M} \leftrightarroweq M \otimes \mathsf{M}'$$

Proof of Proposition 6.30 (1/2)

As usual, assume $\mathsf{M} = \langle \mathsf{S}, \sim, \mathsf{pre} \rangle$, $\mathsf{M}' = \langle \mathsf{S}', \sim', \mathsf{pre}' \rangle$ and $\mathfrak{E} : \mathsf{M} \rightleftarrows \mathsf{M}'$.

Proof of Proposition 6.30 (1/2)

As usual, assume $\mathsf{M} = \langle \mathsf{S}, \sim, \mathsf{pre} \rangle$, $\mathsf{M}' = \langle \mathsf{S}', \sim', \mathsf{pre}' \rangle$ and $\mathfrak{E} : \mathsf{M} \rightleftarrows \mathsf{M}'$.
We define

$$\mathfrak{R} := \{((s, \mathsf{s}), (s', \mathsf{s}')) \in S_{\otimes \mathsf{M}} \times S_{\otimes \mathsf{M}'} \mid s = s' \ \& \ \mathsf{s} \, \mathfrak{E} \, \mathsf{s}'\}$$

i.e.

$$(s, \mathsf{s}) \, \mathfrak{R} \, (s', \mathsf{s}') \iff s = s' \ \& \ \mathsf{s} \, \mathfrak{E} \, \mathsf{s}'$$

where

$$S_{\otimes \mathsf{M}} := \{(s, \mathsf{s}) \in S \times \mathsf{S} \mid M, s \models \mathsf{pre}(\mathsf{s})\}$$

and similarly for $S_{\otimes \mathsf{M}'}$.

Let's assume $a \in A$ and $(s, \mathsf{s}) \, \mathfrak{R} \, (s', \mathsf{s}')$

Proof of Proposition 6.30 (1/2)

As usual, assume $\mathsf{M} = \langle \mathsf{S}, \sim, \mathsf{pre} \rangle$, $\mathsf{M}' = \langle \mathsf{S}', \sim', \mathsf{pre}' \rangle$ and $\mathfrak{E} : \mathsf{M} \rightleftarrows \mathsf{M}'$. We define

$$\mathfrak{R} := \{((s, \mathsf{s}), (s', \mathsf{s}')) \in S_{\otimes \mathsf{M}} \times S_{\otimes \mathsf{M}'} \mid s = s' \;\&\; \mathsf{s}\,\mathfrak{E}\,\mathsf{s}'\}$$

i.e.

$$(s, \mathsf{s})\,\mathfrak{R}\,(s', \mathsf{s}') \iff s = s' \;\&\; \mathsf{s}\,\mathfrak{E}\,\mathsf{s}'$$

where

$$S_{\otimes \mathsf{M}} := \{(s, \mathsf{s}) \in S \times \mathsf{S} \mid M, s \models \mathsf{pre}(\mathsf{s})\}$$

and similarly for $S_{\otimes \mathsf{M}'}$.

Let's assume $a \in A$ and $(s, \mathsf{s})\,\mathfrak{R}\,(s', \mathsf{s}')$; i.e.

$$s = s' \;\&\; M, s \models \mathsf{pre}(\mathsf{s}) \wedge \mathsf{pre}'(\mathsf{s}') \;\&\; \mathsf{s}\,\mathfrak{E}\,\mathsf{s}'$$

Proof of Proposition 6.30 (1/2)

As usual, assume $\mathsf{M} = \langle \mathsf{S}, \sim, \mathsf{pre} \rangle$, $\mathsf{M}' = \langle \mathsf{S}', \sim', \mathsf{pre}' \rangle$ and $\mathfrak{E} : \mathsf{M} \rightleftarrows \mathsf{M}'$. We define

$$\mathfrak{R} := \{((s, \mathsf{s}), (s', \mathsf{s}')) \in S_{\otimes \mathsf{M}} \times S_{\otimes \mathsf{M}'} \mid s = s' \ \& \ \mathsf{s} \, \mathfrak{E} \, \mathsf{s}'\}$$

i.e.

$$(s, \mathsf{s}) \, \mathfrak{R} \, (s', \mathsf{s}') \iff s = s' \ \& \ \mathsf{s} \, \mathfrak{E} \, \mathsf{s}'$$

where

$$S_{\otimes \mathsf{M}} := \{(s, \mathsf{s}) \in S \times \mathsf{S} \mid M, s \models \mathsf{pre}(\mathsf{s})\}$$

and similarly for $S_{\otimes \mathsf{M}'}$.

Let's assume $a \in A$ and $(s, \mathsf{s}) \, \mathfrak{R} \, (s', \mathsf{s}')$; i.e.

$$s = s' \ \& \ M, s \models \mathsf{pre}(\mathsf{s}) \wedge \mathsf{pre}'(\mathsf{s}') \ \& \ \mathsf{s} \, \mathfrak{E} \, \mathsf{s}'$$

atoms $(s, \mathsf{s}) \in V_{\otimes \mathsf{M}}(p)$

Proof of Proposition 6.30 (1/2)

As usual, assume $\mathsf{M} = \langle \mathsf{S}, \sim, \mathsf{pre} \rangle$, $\mathsf{M}' = \langle \mathsf{S}', \sim', \mathsf{pre}' \rangle$ and $\mathfrak{E} : \mathsf{M} \rightleftarrows \mathsf{M}'$.
We define

$$\mathfrak{R} := \{((s, \mathsf{s}), (s', \mathsf{s}')) \in S_{\otimes \mathsf{M}} \times S_{\otimes \mathsf{M}'} \mid s = s' \ \& \ \mathsf{s} \, \mathfrak{E} \, \mathsf{s}'\}$$

i.e.

$$(s, \mathsf{s}) \, \mathfrak{R} \, (s', \mathsf{s}') \iff s = s' \ \& \ \mathsf{s} \, \mathfrak{E} \, \mathsf{s}'$$

where

$$S_{\otimes \mathsf{M}} := \{(s, \mathsf{s}) \in S \times \mathsf{S} \mid M, s \models \mathsf{pre}(\mathsf{s})\}$$

and similarly for $S_{\otimes \mathsf{M}'}$.

Let's assume $a \in A$ and $(s, \mathsf{s}) \, \mathfrak{R} \, (s', \mathsf{s}')$; i.e.

$$s = s' \ \& \ M, s \models \mathsf{pre}(\mathsf{s}) \wedge \mathsf{pre}'(\mathsf{s}') \ \& \ \mathsf{s} \, \mathfrak{E} \, \mathsf{s}'$$

atoms $(s, \mathsf{s}) \in V_{\otimes \mathsf{M}}(p) \iff s \in V(p)$

Proof of Proposition 6.30 (1/2)

As usual, assume $\mathsf{M} = \langle \mathsf{S}, \sim, \mathsf{pre} \rangle$, $\mathsf{M}' = \langle \mathsf{S}', \sim', \mathsf{pre}' \rangle$ and $\mathfrak{E} : \mathsf{M} \rightleftarrows \mathsf{M}'$. We define

$$\mathfrak{R} := \{((s, \mathsf{s}), (s', \mathsf{s}')) \in S_{\otimes \mathsf{M}} \times S_{\otimes \mathsf{M}'} \mid s = s' \;\&\; \mathsf{s} \,\mathfrak{E}\, \mathsf{s}'\}$$

i.e.

$$(s, \mathsf{s}) \,\mathfrak{R}\, (s', \mathsf{s}') \iff s = s' \;\&\; \mathsf{s} \,\mathfrak{E}\, \mathsf{s}'$$

where

$$S_{\otimes \mathsf{M}} := \{(s, \mathsf{s}) \in S \times \mathsf{S} \mid M, s \models \mathsf{pre}(\mathsf{s})\}$$

and similarly for $S_{\otimes \mathsf{M}'}$.

Let's assume $a \in A$ and $(s, \mathsf{s}) \,\mathfrak{R}\, (s', \mathsf{s}')$; i.e.

$$s = s' \;\&\; M, s \models \mathsf{pre}(\mathsf{s}) \wedge \mathsf{pre}'(\mathsf{s}') \;\&\; \mathsf{s} \,\mathfrak{E}\, \mathsf{s}'$$

atoms $(s, \mathsf{s}) \in V_{\otimes \mathsf{M}}(p) \iff s \in V(p) \iff (s', \mathsf{s}') \in V_{\otimes \mathsf{M}'}(p)$, for any $p \in P$

Proof of Proposition 6.30 (2/2)

forth Let $(s, \mathsf{s}) \sim_a (t, \mathsf{t})$.

Proof of Proposition 6.30 (2/2)

forth Let $(s, \mathsf{s}) \sim_a (t, \mathsf{t})$.
It suffices to show that there is $(t', \mathsf{t}') \in \mathsf{S}_{\otimes \mathsf{M}'}$, s.t. $(t, \mathsf{t}) \, \mathfrak{R} \, (t', \mathsf{t}')$ and $(s', \mathsf{s}') \sim_a' (t', \mathsf{t}')$.

Proof of Proposition 6.30 (2/2)

forth Let $(s, \mathsf{s}) \sim_a (t, \mathsf{t})$.
It suffices to show that there is $(t', \mathsf{t}') \in \mathsf{S}_{\otimes \mathsf{M}'}$, s.t. $(t, \mathsf{t}) \, \mathfrak{R} \, (t', \mathsf{t}')$ and $(s', \mathsf{s}') \sim'_a (t', \mathsf{t}')$.
Equivalently, it suffices to show that there is $\mathsf{t}' \in \mathsf{S}'$ s.t.

$$M, t \models \mathsf{pre}'(\mathsf{t}') \;\; \& \; \mathsf{t} \, \mathfrak{E} \, \mathsf{t}' \; \& \; \mathsf{s}' \sim'_a \mathsf{t}'$$

Proof of Proposition 6.30 (2/2)

forth Let $(s, \mathsf{s}) \sim_a (t, \mathsf{t})$.
It suffices to show that there is $(t', \mathsf{t}') \in \mathsf{S}_{\otimes \mathsf{M}'}$, s.t.
$(t, \mathsf{t}) \mathfrak{R} (t', \mathsf{t}')$ and $(s', \mathsf{s}') \sim'_a (t', \mathsf{t}')$.
Equivalently, it suffices to show that there is $\mathsf{t}' \in \mathsf{S}'$ s.t.

$$M, t \models \mathsf{pre}'(\mathsf{t}') \ \& \ \mathsf{t} \, \mathfrak{E} \, \mathsf{t}' \ \& \ \mathsf{s}' \sim'_a \mathsf{t}'$$

By $\mathsf{s} \, \mathfrak{E} \, \mathsf{s}'$, $(s, \mathsf{s}) \sim_a (t, \mathsf{t})$ (which implies $\mathsf{s} \sim_a \mathsf{t}$) and **Forth** for emulation $\mathfrak{E}$, we have that there are $\mathsf{t}'_1, \ldots, \mathsf{t}'_n \in \mathsf{S}'$ s.t. for all $i \in [n]$, $\mathsf{t} \, \mathfrak{E} \, \mathsf{t}'_i$ and $\mathsf{s}' \sim'_a \mathsf{t}'_i$ and s.t.
$\mathsf{pre}(\mathsf{t}) \models \mathsf{pre}'\left(\mathsf{t}'_1\right) \vee \cdots \vee \mathsf{pre}'(\mathsf{t}'_n)$.

Proof of Proposition 6.30 (2/2)

forth Let $(s, \mathsf{s}) \sim_a (t, \mathsf{t})$.
It suffices to show that there is $(t', \mathsf{t}') \in \mathsf{S}_{\otimes \mathsf{M}'}$, s.t. $(t, \mathsf{t}) \mathfrak{R} (t', \mathsf{t}')$ and $(s', \mathsf{s}') \sim'_a (t', \mathsf{t}')$.
Equivalently, it suffices to show that there is $\mathsf{t}' \in \mathsf{S}'$ s.t.

$$M, t \models \mathsf{pre}'(\mathsf{t}') \ \& \ \mathsf{t}\, \mathfrak{E}\, \mathsf{t}' \ \& \ \mathsf{s}' \sim'_a \mathsf{t}'$$

By $\mathsf{s}\, \mathfrak{E}\, \mathsf{s}'$, $(s, \mathsf{s}) \sim_a (t, \mathsf{t})$ (which implies $\mathsf{s} \sim_a \mathsf{t}$) and **Forth** for emulation $\mathfrak{E}$, we have that there are $\mathsf{t}'_1, \ldots, \mathsf{t}'_n \in \mathsf{S}'$ s.t. for all $i \in [n]$, $\mathsf{t}\, \mathfrak{E}\, \mathsf{t}'_i$ and $\mathsf{s}' \sim'_a \mathsf{t}'_i$ and s.t. $\mathsf{pre}(\mathsf{t}) \models \mathsf{pre}'\big(\mathsf{t}'_1\big) \vee \cdots \vee \mathsf{pre}'(\mathsf{t}'_n)$.
But as $(t, \mathsf{t}) \in S_{\otimes \mathsf{M}}$ we have that $M, t \models \mathsf{pre}(\mathsf{t})$ and thus $M, t \models \mathsf{pre}'\big(\mathsf{t}'_1\big) \vee \cdots \vee \mathsf{pre}'(\mathsf{t}'_n)$.

Proof of Proposition 6.30 (2/2)

forth Let $(s, \mathsf{s}) \sim_a (t, \mathsf{t})$.
It suffices to show that there is $(t', \mathsf{t}') \in \mathsf{S}_{\otimes \mathsf{M}'}$, s.t.
$(t, \mathsf{t}) \,\mathfrak{R}\, (t', \mathsf{t}')$ and $(s', \mathsf{s}') \sim'_a (t', \mathsf{t}')$.
Equivalently, it suffices to show that there is $\mathsf{t}' \in \mathsf{S}'$ s.t.

$$M, t \models \mathsf{pre}'(\mathsf{t}') \;\&\; \mathsf{t} \,\mathfrak{E}\, \mathsf{t}' \;\&\; \mathsf{s}' \sim'_a \mathsf{t}'$$

By $\mathsf{s} \,\mathfrak{E}\, \mathsf{s}'$, $(s, \mathsf{s}) \sim_a (t, \mathsf{t})$ (which implies $\mathsf{s} \sim_a \mathsf{t}$) and **Forth** for emulation $\mathfrak{E}$, we have that there are $\mathsf{t}'_1, \ldots, \mathsf{t}'_n \in \mathsf{S}'$ s.t. for all $i \in [n]$, $\mathsf{t} \,\mathfrak{E}\, \mathsf{t}'_i$ and $\mathsf{s}' \sim'_a \mathsf{t}'_i$ and s.t.
$\mathsf{pre}(\mathsf{t}) \models \mathsf{pre}'\left(\mathsf{t}'_1\right) \vee \cdots \vee \mathsf{pre}'(\mathsf{t}'_n)$.
But as $(t, \mathsf{t}) \in S_{\otimes \mathsf{M}}$ we have that $M, t \models \mathsf{pre}(\mathsf{t})$ and thus
$M, t \models \mathsf{pre}'\left(\mathsf{t}'_1\right) \vee \cdots \vee \mathsf{pre}'(\mathsf{t}'_n)$.
Therefore, there is some $i \in [n]$ s.t.

$$M, t \models \mathsf{pre}'(\mathsf{t}'_i) \;\&\; \mathsf{t} \,\mathfrak{E}\, \mathsf{t}'_i \;\&\; \mathsf{s}' \sim'_a \mathsf{t}'_i$$

Proof of Proposition 6.30 (2/2)

forth Let $(s, \mathsf{s}) \sim_a (t, \mathsf{t})$.
It suffices to show that there is $(t', \mathsf{t}') \in \mathsf{S}_{\otimes \mathsf{M}'}$, s.t. $(t, \mathsf{t}) \mathfrak{R} (t', \mathsf{t}')$ and $(s', \mathsf{s}') \sim'_a (t', \mathsf{t}')$.
Equivalently, it suffices to show that there is $\mathsf{t}' \in \mathsf{S}'$ s.t.

$$M, t \models \mathsf{pre}'(\mathsf{t}') \;\&\; \mathsf{t} \,\mathfrak{E}\, \mathsf{t}' \;\&\; \mathsf{s}' \sim'_a \mathsf{t}'$$

By $\mathsf{s} \,\mathfrak{E}\, \mathsf{s}'$, $(s, \mathsf{s}) \sim_a (t, \mathsf{t})$ (which implies $\mathsf{s} \sim_a \mathsf{t}$) and **Forth** for emulation $\mathfrak{E}$, we have that there are $\mathsf{t}'_1, \ldots, \mathsf{t}'_n \in \mathsf{S}'$ s.t. for all $i \in [n]$, $\mathsf{t} \,\mathfrak{E}\, \mathsf{t}'_i$ and $\mathsf{s}' \sim'_a \mathsf{t}'_i$ and s.t. $\mathsf{pre}(\mathsf{t}) \models \mathsf{pre}'\left(\mathsf{t}'_1\right) \vee \cdots \vee \mathsf{pre}'(\mathsf{t}'_n)$.
But as $(t, \mathsf{t}) \in S_{\otimes \mathsf{M}}$ we have that $M, t \models \mathsf{pre}(\mathsf{t})$ and thus $M, t \models \mathsf{pre}'\left(\mathsf{t}'_1\right) \vee \cdots \vee \mathsf{pre}'(\mathsf{t}'_n)$.
Therefore, there is some $i \in [n]$ s.t.

$$M, t \models \mathsf{pre}'(\mathsf{t}'_i) \;\&\; \mathsf{t} \,\mathfrak{E}\, \mathsf{t}'_i \;\&\; \mathsf{s}' \sim'_a \mathsf{t}'_i$$

back Similarly with **forth**. □

Proposition 6.30 for pointed Action Models?

The obvious 'pointed' version of Proposition 6.30 does not hold!

Proposition 6.30 for pointed Action Models?

The obvious 'pointed' version of Proposition 6.30 does not hold!

Given an epistemic state (M, s) and action models $(\mathsf{M}, \mathsf{s}) \rightleftarrows (\mathsf{M}', \mathsf{s}')$ and such that (M, s) is executable in (M, s), then $(M \otimes \mathsf{M}, (s, \mathsf{s}))$ may not be bisimilar to $(M \otimes \mathsf{M}', (s, \mathsf{s}'))$.

Proposition 6.30 for pointed Action Models?

The obvious 'pointed' version of Proposition 6.30 does not hold!

Given an epistemic state (M, s) and action models $(\mathsf{M}, \mathsf{s}) \rightleftarrows (\mathsf{M}', \mathsf{s}')$ and such that (M, s) is executable in (M, s), then $(M \otimes \mathsf{M}, (s, \mathsf{s}))$ may not be bisimilar to $(M \otimes \mathsf{M}', (s, \mathsf{s}'))$.

This is because $\mathsf{pre}'(\mathsf{s}')$ may not be true in (M, s).

Proposition 6.30 for pointed Action Models?

The obvious 'pointed' version of Proposition 6.30 does not hold!

Given an epistemic state (M, s) and action models $(\mathsf{M}, \mathsf{s}) \rightleftarrows (\mathsf{M}', \mathsf{s}')$ and such that (M, s) is executable in (M, s), then $(M \otimes \mathsf{M}, (s, \mathsf{s}))$ may not be bisimilar to $(M \otimes \mathsf{M}', (s, \mathsf{s}'))$.

This is because $\mathsf{pre}'(\mathsf{s}')$ may not be true in (M, s).

Although there must be a $\mathsf{s}'_i \in \mathsf{S}'$ among the 'alternatives' for s' with $\mathsf{pre}(\mathsf{s}) \models \mathsf{pre}'\left(\mathsf{s}'_1\right) \vee \dots \vee \mathsf{pre}'(\mathsf{s}'_n) \vee \dots$, s.t. this s'_n fulfils the role required for $(M \otimes \mathsf{M}, (s, \mathsf{s}'_n))$

Axiomatization for Action Model Logic

The axiom system for Action Model logic is denoted as **AMC**.

$$\mathbf{AMC} = \mathbf{S5C} + \text{axioms for action models}$$

S5C Axiom System

Axiom Schemes

$K_a(\varphi \to \psi) \to K_a\varphi \to K_a\psi$	distribution of K_a over $\to$
$K_a\varphi \to \varphi$	truth
$K_a\varphi \to K_aK_a\varphi$	positive introspection
$\neg K_a\varphi \to K_a\neg K_a\varphi$	negative introspection
$C_B(\varphi \to \psi) \to C_B\varphi \to C_B\psi$	distribution of C_B over $\to$
$C_B\varphi \to (\varphi \wedge E_BC_B\varphi)$	mix
$C_B(\varphi \to E_B\varphi) \to \varphi \to C_B\varphi$	induction axiom

Rules of inference

From φ and $\varphi \to \psi$, infer ψ	modus ponens
From φ, infer $K_a\varphi$	necessitation of K_a
From φ, infer $C_B\varphi$	necessitation of C_B

Axiomatization for Action Model Logic

Axioms for Action Models

Axiom Schemes

$[\mathsf{M},\mathsf{s}]\, p \leftrightarrow (\mathsf{pre}(\mathsf{s}) \to p)$	atomic permanence
$[\mathsf{M},\mathsf{s}]\, \neg\varphi \leftrightarrow (\mathsf{pre}(\mathsf{s}) \to \neg\, [\mathsf{M},\mathsf{s}]\, \varphi)$	action and negation
$[\mathsf{M},\mathsf{s}]\, (\varphi \wedge \psi) \leftrightarrow [\mathsf{M},\mathsf{s}]\, \varphi \wedge [\mathsf{M},\mathsf{s}]\, \psi$	action and conjunction
$[\mathsf{M},\mathsf{s}]\, K_a\varphi \leftrightarrow \left(\mathsf{pre}(\mathsf{s}) \to \bigwedge_{\mathsf{s}\sim_a \mathsf{t}} K_a\, [\mathsf{M},\mathsf{t}]\, \varphi\right)$	action and knowledge
$[\mathsf{M},\mathsf{s}]\, [\mathsf{M}',\mathsf{s}']\, \varphi \leftrightarrow [(\mathsf{M},\mathsf{s})\, ;\, (\mathsf{M}',\mathsf{s}')]\, \varphi$	action composition
$[\alpha \cup \beta]\, \varphi \leftrightarrow [\alpha]\, \varphi \wedge [\beta]\, \varphi$	non-deterministic choice

Rules of inference

From φ, infer $[\mathsf{M},\mathsf{s}]\, \varphi$	necessitation of (M,s)
Given (M,s), and χ_t for all $\mathsf{t} \sim_B \mathsf{s}$. If for all $a \in B$ and $\mathsf{u} \sim_a \mathsf{t} : \chi_\mathsf{t} \to [\mathsf{M},\mathsf{t}]\, \varphi$ and $(\chi_\mathsf{t} \wedge \mathsf{pre}(\mathsf{t})) \to K_a\chi_\mathsf{u}$, then $\chi_\mathsf{s} \to [\mathsf{M},\mathsf{s}]\, C_B\varphi$.	action and comm. knowl.

Soundness of AMC

Theorem (Propositions 6.9, 6.11, 6.32-6.37)

Axiom system **AMC** *is sound with respect of* AMC; *i.e. for any* $\varphi \in \mathcal{L}^{\mathbf{stat}}_{KC\otimes}(A, P)$

$$\mathbf{AMC} \vdash \varphi \implies AMC \models \varphi$$

Soundness of **AMC**

Proof of Atomic permanence $[\mathsf{M}, \mathsf{s}]\, p \leftrightarrow (\mathsf{pre}(\mathsf{s}) \rightarrow p)$

Soundness of **AMC**

Proof of Atomic permanence $[\mathsf{M}, \mathsf{s}]\, p \leftrightarrow (\mathsf{pre}(\mathsf{s}) \rightarrow p)$

Let arbitrary epistemic state (M, t) s.t. $M, t \models [\mathsf{M}, \mathsf{s}]\, p$.

Soundness of **AMC**

Proof of Atomic permanence $[\mathsf{M},\mathsf{s}]\, p \leftrightarrow (\mathsf{pre}(\mathsf{s}) \to p)$

Let arbitrary epistemic state (M, t) s.t. $M, t \models [\mathsf{M},\mathsf{s}]\, p$.

Equivalently, for any epistemic state (M', t') s.t. $(M, t)\, [\![\mathsf{M},\mathsf{s}]\!]\, (M', t')$, we have $M', t' \models p$.

Soundness of **AMC**

Proof of Atomic permanence $[\mathsf{M},\mathsf{s}]\,p \leftrightarrow (\mathsf{pre}(\mathsf{s}) \rightarrow p)$

Let arbitrary epistemic state (M, t) s.t. $M, t \models [\mathsf{M},\mathsf{s}]\,p$.

Equivalently, for any epistemic state (M', t') s.t. $(M, t)\,[\![\mathsf{M},\mathsf{s}]\!]\,(M', t')$, we have $M', t' \models p$.

Equivalently, if $M, t \models \mathsf{pre}(\mathsf{s})$, then
$M \otimes \mathsf{M}, (t, \mathsf{s}) \models p$

Soundness of AMC

Proof of Atomic permanence $[\mathsf{M}, \mathsf{s}]\, p \leftrightarrow (\mathsf{pre}(\mathsf{s}) \rightarrow p)$

Let arbitrary epistemic state (M, t) s.t. $M, t \models [\mathsf{M}, \mathsf{s}]\, p$.

Equivalently, for any epistemic state (M', t') s.t. $(M, t)\, [\![\mathsf{M}, \mathsf{s}]\!]\, (M', t')$, we have $M', t' \models p$.

Equivalently, if $M, t \models \mathsf{pre}(\mathsf{s})$, then
$M \otimes \mathsf{M}, (t, \mathsf{s}) \models p \Leftrightarrow (t, \mathsf{s}) \in V_{\otimes \mathsf{M}}(p)$

Soundness of AMC

Proof of Atomic permanence $[\mathsf{M}, \mathsf{s}]\, p \leftrightarrow (\mathsf{pre}(\mathsf{s}) \rightarrow p)$

Let arbitrary epistemic state (M, t) s.t. $M, t \models [\mathsf{M}, \mathsf{s}]\, p$.

Equivalently, for any epistemic state (M', t') s.t. $(M, t)\, [\![\mathsf{M}, \mathsf{s}]\!]\, (M', t')$, we have $M', t' \models p$.

Equivalently, if $M, t \models \mathsf{pre}(\mathsf{s})$, then
$M \otimes \mathsf{M}, (t, \mathsf{s}) \models p \Leftrightarrow (t, \mathsf{s}) \in V_{\otimes \mathsf{M}}(p) \;\Leftrightarrow t \in V(p)$

Soundness of **AMC**

Proof of Atomic permanence $[\mathsf{M}, \mathsf{s}]\, p \leftrightarrow (\mathsf{pre}(\mathsf{s}) \rightarrow p)$

Let arbitrary epistemic state (M, t) s.t. $M, t \models [\mathsf{M}, \mathsf{s}]\, p$.

Equivalently, for any epistemic state (M', t') s.t. $(M, t)\, [\![\mathsf{M}, \mathsf{s}]\!]\, (M', t')$, we have $M', t' \models p$.

Equivalently, if $M, t \models \mathsf{pre}(\mathsf{s})$, then
$M \otimes \mathsf{M}, (t, \mathsf{s}) \models p \Leftrightarrow (t, \mathsf{s}) \in V_{\otimes \mathsf{M}}(p) \;\Leftrightarrow t \in V(p) \;\Leftrightarrow M, t \models p$.

Soundness of AMC

Proof of Atomic permanence $[\mathsf{M}, \mathsf{s}]\, p \leftrightarrow (\mathsf{pre}(\mathsf{s}) \rightarrow p)$

Let arbitrary epistemic state (M, t) s.t. $M, t \models [\mathsf{M}, \mathsf{s}]\, p$.

Equivalently, for any epistemic state (M', t') s.t. $(M, t)\, [\![\mathsf{M}, \mathsf{s}]\!]\, (M', t')$, we have $M', t' \models p$.

Equivalently, if $M, t \models \mathsf{pre}(\mathsf{s})$, then
$M \otimes \mathsf{M}, (t, \mathsf{s}) \models p \Leftrightarrow (t, \mathsf{s}) \in V_{\otimes \mathsf{M}}(p) \;\Leftrightarrow t \in V(p) \;\Leftrightarrow M, t \models p$.

Equivalently, $M, t \models \mathsf{pre}(\mathsf{s}) \rightarrow p$

□

Soundness of **AMC**

Proof of Action and Common Knowledge Axiom (1/3)

Soundness of AMC

Proof of Action and Common Knowledge Axiom (1/3)

Let $\forall \mathsf{t} \sim_B \mathsf{s}\ \forall a \in B\ \forall \mathsf{u} \sim_a \mathsf{t}$

$$\models \chi_{\mathsf{t}} \rightarrow [\mathsf{M}, \mathsf{t}]\,\varphi \quad \& \quad \models \chi_{\mathsf{t}} \wedge \mathsf{pre}(\mathsf{t}) \rightarrow K_a \chi_{\mathsf{u}}$$

Soundness of AMC

Proof of Action and Common Knowledge Axiom (1/3)

Let $\forall \mathsf{t} \sim_B \mathsf{s}\ \forall a \in B\ \forall \mathsf{u} \sim_a \mathsf{t}$

$$\models \chi_\mathsf{t} \rightarrow [\mathsf{M}, \mathsf{t}]\,\varphi \quad \& \quad \models \chi_\mathsf{t} \wedge \mathsf{pre}(\mathsf{t}) \rightarrow K_a \chi_\mathsf{u}$$

We want to show that $\models \chi_\mathsf{s} \rightarrow [\mathsf{M}, \mathsf{s}]\, C_B \varphi$.

Soundness of **AMC**

Proof of Action and Common Knowledge Axiom (1/3)

Let $\forall \mathsf{t} \sim_B \mathsf{s}\ \forall a \in B\ \forall \mathsf{u} \sim_a \mathsf{t}$

$$\models \chi_{\mathsf{t}} \to [\mathsf{M}, \mathsf{t}]\, \varphi \quad \& \quad \models \chi_{\mathsf{t}} \wedge \mathsf{pre}(\mathsf{t}) \to K_a \chi_{\mathsf{u}}$$

We want to show that $\models \chi_{\mathsf{s}} \to [\mathsf{M}, \mathsf{s}]\, C_B \varphi$.

Let arbitrary $S5$ epistemic state (M, s) s.t. $M, s \models \chi_{\mathsf{s}} \wedge \mathsf{pre}(\mathsf{s})$.
It suffices to show that $M \otimes \mathsf{M}, (s, \mathsf{s}) \models C_B \varphi$.

Soundness of AMC

Proof of Action and Common Knowledge Axiom (1/3)

Let $\forall \mathsf{t} \sim_B \mathsf{s}\ \forall a \in B\ \forall \mathsf{u} \sim_a \mathsf{t}$

$$\models \chi_{\mathsf{t}} \to [\mathsf{M}, \mathsf{t}]\, \varphi \quad \& \quad \models \chi_{\mathsf{t}} \wedge \mathsf{pre}(\mathsf{t}) \to K_a \chi_{\mathsf{u}}$$

We want to show that $\models \chi_{\mathsf{s}} \to [\mathsf{M}, \mathsf{s}]\, C_B \varphi$.

Let arbitrary $S5$ epistemic state (M, s) s.t. $M, s \models \chi_{\mathsf{s}} \wedge \mathsf{pre}(\mathsf{s})$.
It suffices to show that $M \otimes \mathsf{M}, (s, \mathsf{s}) \models C_B \varphi$.

By Remark (2.29), it suffices to to show that

$$\forall n \in \mathbb{N}\ \forall (t, \mathsf{t}) \sim^{n}_{\otimes \mathsf{M}; E_B} (s, \mathsf{s}) \quad M \otimes \mathsf{M}, (t, \mathsf{t}) \models \varphi$$

Soundness of AMC

Proof of Action and Common Knowledge Axiom (1/3)

Let $\forall \mathsf{t} \sim_B \mathsf{s}\ \forall a \in B\ \forall \mathsf{u} \sim_a \mathsf{t}$

$$\models \chi_{\mathsf{t}} \rightarrow [\mathsf{M}, \mathsf{t}]\, \varphi \quad \& \quad \models \chi_{\mathsf{t}} \wedge \mathsf{pre}(\mathsf{t}) \rightarrow K_a \chi_{\mathsf{u}}$$

We want to show that $\models \chi_{\mathsf{s}} \rightarrow [\mathsf{M}, \mathsf{s}]\, C_B \varphi$.

Let arbitrary $S5$ epistemic state (M, s) s.t. $M, s \models \chi_{\mathsf{s}} \wedge \mathsf{pre}(\mathsf{s})$.
It suffices to show that $M \otimes \mathsf{M}, (s, \mathsf{s}) \models C_B \varphi$.

By Remark (2.29), it suffices to to show that

$$\forall n \in \mathbb{N}\ \forall\, (t, \mathsf{t}) \sim^n_{\otimes \mathsf{M}; E_B} (s, \mathsf{s}) \quad M \otimes \mathsf{M}, (t, \mathsf{t}) \models \varphi$$

i.e. for any epistemic point (t, t) reachable from (s, s) through $\sim_{\otimes \mathsf{M}; E_B}$
$M \otimes \mathsf{M}, (t, \mathsf{t}) \models \varphi$.

Soundness of AMC

Proof of Action and Common Knowledge Axiom (2/3)

We will prove, by induction on n, the stronger statement, that $\forall n \in \mathbb{N}\ \forall\, (t, \mathrm{t}) \sim^{n}_{\otimes \mathrm{M}; E_B} (s, \mathrm{s})$

$$M \otimes \mathrm{M}, (t, \mathrm{t}) \models \varphi \quad \& \quad M, t \models \chi_{\mathrm{t}}$$

Soundness of AMC

Proof of Action and Common Knowledge Axiom (2/3)

We will prove, by induction on n, the stronger statement, that $\forall n \in \mathbb{N}\ \forall\, (t, \mathsf{t}) \sim^{n}_{\otimes \mathsf{M}; E_B} (s, \mathsf{s})$

$$M \otimes \mathsf{M}, (t, \mathsf{t}) \models \varphi \quad \& \quad M, t \models \chi_{\mathsf{t}}$$

- $\mathbf{n = 0}$

Soundness of **AMC**

Proof of Action and Common Knowledge Axiom (2/3)

We will prove, by induction on n, the stronger statement, that $\forall n \in \mathbb{N}\ \forall\, (t, \mathrm{t}) \sim^{n}_{\otimes \mathrm{M}; E_B} (s, \mathrm{s})$

$$M \otimes \mathrm{M}, (t, \mathrm{t}) \models \varphi \quad \& \quad M, t \models \chi_{\mathrm{t}}$$

- **n = 0** Then $(t, \mathrm{t}) = (s, \mathrm{s})$ and $M, s \models \mathrm{pre}(\mathrm{s})$

Soundness of AMC

Proof of Action and Common Knowledge Axiom (2/3)

We will prove, by induction on n, the stronger statement, that $\forall n \in \mathbb{N}\ \forall\, (t, \mathrm{t}) \sim^{n}_{\otimes \mathrm{M}; E_B} (s, \mathrm{s})$

$$M \otimes \mathrm{M}, (t, \mathrm{t}) \models \varphi \quad \& \quad M, t \models \chi_{\mathrm{t}}$$

- **n = 0** Then $(t, \mathrm{t}) = (s, \mathrm{s})$ and
  $M, s \models \mathrm{pre}(\mathrm{s})$ $(s, \mathrm{s}) \in S_{\otimes \mathrm{M}}$

Soundness of AMC

Proof of Action and Common Knowledge Axiom (2/3)

We will prove, by induction on n, the stronger statement, that $\forall n \in \mathbb{N}\ \forall (t, \mathrm{t}) \sim^{n}_{\otimes \mathrm{M}; E_B} (s, \mathrm{s})$

$$M \otimes \mathrm{M}, (t, \mathrm{t}) \models \varphi \quad \& \quad M, t \models \chi_{\mathrm{t}}$$

- $\mathbf{n = 0}$ Then $(t, \mathrm{t}) = (s, \mathrm{s})$ and
  $M, s \models \mathrm{pre}(\mathrm{s})$ $(s, \mathrm{s}) \in S_{\otimes \mathrm{M}}$
  $M, s \models \chi_{\mathrm{s}}$

Soundness of AMC

Proof of Action and Common Knowledge Axiom (2/3)

We will prove, by induction on n, the stronger statement, that $\forall n \in \mathbb{N}\ \forall (t, \mathrm{t}) \sim^{n}_{\otimes \mathrm{M}; E_B} (s, \mathrm{s})$

$$M \otimes \mathrm{M}, (t, \mathrm{t}) \models \varphi \quad \& \quad M, t \models \chi_{\mathrm{t}}$$

- $\mathbf{n = 0}$ Then $(t, \mathrm{t}) = (s, \mathrm{s})$ and
  $M, s \models \mathrm{pre}(\mathrm{s})$ $(s, \mathrm{s}) \in S_{\otimes \mathrm{M}}$
  $M, s \models \chi_{\mathrm{s}}$ by hypothesis

Soundness of AMC

Proof of Action and Common Knowledge Axiom (2/3)

We will prove, by induction on n, the stronger statement, that $\forall n \in \mathbb{N}\ \forall (t, \mathsf{t}) \sim^{n}_{\otimes \mathsf{M}; E_B} (s, \mathsf{s})$

$$M \otimes \mathsf{M}, (t, \mathsf{t}) \models \varphi \quad \& \quad M, t \models \chi_{\mathsf{t}}$$

- **n = 0** Then $(t, \mathsf{t}) = (s, \mathsf{s})$ and
  $M, s \models \mathsf{pre}(\mathsf{s})$ $\qquad (s, \mathsf{s}) \in S_{\otimes \mathsf{M}}$
  $M, s \models \chi_{\mathsf{s}}$ $\qquad$ by hypothesis
  Thus by $\models \chi_{\mathsf{s}} \to [\mathsf{M}, \mathsf{s}]\varphi$, we have $M \otimes \mathsf{M}, (s, \mathsf{s}) \models \varphi$ and $M, s \models \chi_{\mathsf{s}}$

Soundness of AMC

Proof of Action and Common Knowledge Axiom (2/3)

We will prove, by induction on n, the stronger statement, that $\forall n \in \mathbb{N}\ \forall (t,\mathsf{t}) \sim^{n}_{\otimes \mathsf{M}; E_B} (s,\mathsf{s})$

$$M \otimes \mathsf{M}, (t,\mathsf{t}) \models \varphi \quad \& \quad M, t \models \chi_{\mathsf{t}}$$

- **n = 0** Then $(t,\mathsf{t}) = (s,\mathsf{s})$ and
  $M, s \models \mathsf{pre}(\mathsf{s})$ $(s,\mathsf{s}) \in S_{\otimes \mathsf{M}}$
  $M, s \models \chi_{\mathsf{s}}$ by hypothesis
  Thus by $\models \chi_{\mathsf{s}} \to [\mathsf{M},\mathsf{s}]\varphi$, we have $M \otimes \mathsf{M}, (s,\mathsf{s}) \models \varphi$ and $M, s \models \chi_{\mathsf{s}}$
- Induction hypothesis (I.H.) Let the statement holds for $n = k \in \mathbb{N}$.

Soundness of AMC

Proof of Action and Common Knowledge Axiom (3/3)

- $\mathbf{n = k + 1}$ Let $(u, \mathsf{u}) \sim^{k+1}_{\otimes \mathsf{M}; E_B} (s, \mathsf{s})$.

Soundness of AMC

Proof of Action and Common Knowledge Axiom (3/3)

- $\mathbf{n} = \mathbf{k} + \mathbf{1}$ Let $(u, \mathsf{u}) \sim^{k+1}_{\otimes \mathsf{M}; E_B} (s, \mathsf{s})$.
  Equivalently, there is $(t, \mathsf{t}) \in S_{\otimes \mathsf{M}}$ s.t. $(s, \mathsf{s}) \sim^{k}_{\otimes \mathsf{M}; E_B} (t, \mathsf{t}) \sim_{\otimes \mathsf{M}; a} (u, \mathsf{u})$, for some $a \in B$.

Soundness of **AMC**

Proof of Action and Common Knowledge Axiom (3/3)

- $\mathbf{n = k + 1}$ Let $(u, \mathsf{u}) \sim^{k+1}_{\otimes \mathsf{M}; E_B} (s, \mathsf{s})$.
  Equivalently, there is $(t, \mathsf{t}) \in S_{\otimes \mathsf{M}}$ s.t. $(s, \mathsf{s}) \sim^{k}_{\otimes \mathsf{M}; E_B} (t, \mathsf{t}) \sim_{\otimes \mathsf{M}; a} (u, \mathsf{u})$, for some $a \in B$.

  $M, t \models \mathsf{pre}(\mathsf{t})$

Soundness of **AMC**

Proof of Action and Common Knowledge Axiom (3/3)

- $\mathbf{n} = \mathbf{k} + \mathbf{1}$ Let $(u, \mathsf{u}) \sim^{k+1}_{\otimes \mathsf{M}; E_B} (s, \mathsf{s})$.
  Equivalently, there is $(t, \mathsf{t}) \in S_{\otimes \mathsf{M}}$ s.t. $(s, \mathsf{s}) \sim^{k}_{\otimes \mathsf{M}; E_B} (t, \mathsf{t}) \sim_{\otimes \mathsf{M}; a} (u, \mathsf{u})$, for some $a \in B$.
  $M, t \models \mathsf{pre}(\mathsf{t})$ $\qquad (t, \mathsf{t}) \in S_{\otimes \mathsf{M}}$

Soundness of AMC

Proof of Action and Common Knowledge Axiom (3/3)

- $\mathbf{n = k + 1}$ Let $(u, \mathsf{u}) \sim^{k+1}_{\otimes\mathsf{M};E_B} (s, \mathsf{s})$.
  Equivalently, there is $(t, \mathsf{t}) \in S_{\otimes\mathsf{M}}$ s.t. $(s, \mathsf{s}) \sim^{k}_{\otimes\mathsf{M};E_B} (t, \mathsf{t}) \sim_{\otimes\mathsf{M};a} (u, \mathsf{u})$, for some $a \in B$.

  $M, t \models \mathsf{pre}(\mathsf{t})$ $\qquad (t, \mathsf{t}) \in S_{\otimes\mathsf{M}}$

  $M, t \models \chi_{\mathsf{t}}$

Soundness of AMC

Proof of Action and Common Knowledge Axiom (3/3)

- $\mathbf{n} = \mathbf{k} + \mathbf{1}$ Let $(u, \mathsf{u}) \sim^{k+1}_{\otimes\mathsf{M};E_B} (s, \mathsf{s})$.
  Equivalently, there is $(t, \mathsf{t}) \in S_{\otimes\mathsf{M}}$ s.t. $(s, \mathsf{s}) \sim^{k}_{\otimes\mathsf{M};E_B} (t, \mathsf{t}) \sim_{\otimes\mathsf{M};a} (u, \mathsf{u})$, for some $a \in B$.

  $M, t \models \mathsf{pre}(\mathsf{t})$ $(t, \mathsf{t}) \in S_{\otimes\mathsf{M}}$
  $M, t \models \chi_{\mathsf{t}}$ by I.H.

Soundness of **AMC**

Proof of Action and Common Knowledge Axiom (3/3)

- $\mathbf{n = k + 1}$ Let $(u, \mathsf{u}) \sim^{k+1}_{\otimes \mathsf{M}; E_B} (s, \mathsf{s})$.
  Equivalently, there is $(t, \mathsf{t}) \in S_{\otimes \mathsf{M}}$ s.t. $(s, \mathsf{s}) \sim^{k}_{\otimes \mathsf{M}; E_B} (t, \mathsf{t}) \sim_{\otimes \mathsf{M}; a} (u, \mathsf{u})$, for some $a \in B$.

  $M, t \models \mathsf{pre}(\mathsf{t})$ $\qquad (t, \mathsf{t}) \in S_{\otimes \mathsf{M}}$
  $M, t \models \chi_{\mathsf{t}}$ $\qquad$ by I.H.

  Thus by $\models \chi_{\mathsf{t}} \wedge \mathsf{pre}(\mathsf{t}) \to K_a \chi_{\mathsf{u}}$, we have $M, t \models K_a \chi_{\mathsf{u}}$ and as $u \sim_a t$ we have $M, u \models \chi_{\mathsf{u}}$.

Soundness of AMC

Proof of Action and Common Knowledge Axiom (3/3)

- $\mathbf{n} = \mathbf{k} + \mathbf{1}$ Let $(u, \mathsf{u}) \sim^{k+1}_{\otimes\mathsf{M};E_B} (s, \mathsf{s})$.
  Equivalently, there is $(t, \mathsf{t}) \in S_{\otimes\mathsf{M}}$ s.t. $(s, \mathsf{s}) \sim^{k}_{\otimes\mathsf{M};E_B} (t, \mathsf{t}) \sim_{\otimes\mathsf{M};a} (u, \mathsf{u})$, for some $a \in B$.

  $M, t \models \mathsf{pre}(\mathsf{t})$ $\quad (t, \mathsf{t}) \in S_{\otimes\mathsf{M}}$
  $M, t \models \chi_{\mathsf{t}}$ $\quad$ by I.H.

  Thus by $\models \chi_{\mathsf{t}} \wedge \mathsf{pre}(\mathsf{t}) \rightarrow K_a\chi_{\mathsf{u}}$, we have $M, t \models K_a\chi_{\mathsf{u}}$ and as $u \sim_a t$ we have $M, u \models \chi_{\mathsf{u}}$.

  $M, u \models \mathsf{pre}(\mathsf{u})$

Soundness of AMC

Proof of Action and Common Knowledge Axiom (3/3)

- **n = k + 1** Let $(u, \mathsf{u}) \sim^{k+1}_{\otimes\mathsf{M};E_B} (s, \mathsf{s})$.
  Equivalently, there is $(t, \mathsf{t}) \in S_{\otimes\mathsf{M}}$ s.t. $(s, \mathsf{s}) \sim^{k}_{\otimes\mathsf{M};E_B} (t, \mathsf{t}) \sim_{\otimes\mathsf{M};a} (u, \mathsf{u})$, for some $a \in B$.

  $M, t \models \mathsf{pre}(\mathsf{t})$ $\qquad (t, \mathsf{t}) \in S_{\otimes\mathsf{M}}$
  $M, t \models \chi_{\mathsf{t}}$ $\qquad$ by I.H.

  Thus by $\models \chi_{\mathsf{t}} \wedge \mathsf{pre}(\mathsf{t}) \to K_a\chi_{\mathsf{u}}$, we have $M, t \models K_a\chi_{\mathsf{u}}$ and as $u \sim_a t$ we have $M, u \models \chi_{\mathsf{u}}$.

  $M, u \models \mathsf{pre}(\mathsf{u})$ $\qquad (u, \mathsf{u}) \in S_{\otimes\mathsf{M}}$

Soundness of AMC

Proof of Action and Common Knowledge Axiom (3/3)

- $\mathbf{n = k + 1}$ Let $(u, \mathsf{u}) \sim^{k+1}_{\otimes \mathsf{M}; E_B} (s, \mathsf{s})$.
  Equivalently, there is $(t, \mathsf{t}) \in S_{\otimes \mathsf{M}}$ s.t. $(s, \mathsf{s}) \sim^{k}_{\otimes \mathsf{M}; E_B} (t, \mathsf{t}) \sim_{\otimes \mathsf{M}; a} (u, \mathsf{u})$, for some $a \in B$.

  $M, t \models \mathsf{pre}(\mathsf{t})$ $\quad (t, \mathsf{t}) \in S_{\otimes \mathsf{M}}$
  $M, t \models \chi_{\mathsf{t}}$ $\quad$ by I.H.

  Thus by $\models \chi_{\mathsf{t}} \wedge \mathsf{pre}(\mathsf{t}) \rightarrow K_a \chi_{\mathsf{u}}$, we have $M, t \models K_a \chi_{\mathsf{u}}$ and as $u \sim_a t$ we have $M, u \models \chi_{\mathsf{u}}$.

  $M, u \models \mathsf{pre}(\mathsf{u})$ $\quad (u, \mathsf{u}) \in S_{\otimes \mathsf{M}}$

  Thus by $M, u \models \chi_{\mathsf{u}}$ and $\models \chi_{\mathsf{u}} \rightarrow [\mathsf{M}, \mathsf{u}] \varphi$ we have $M \otimes \mathsf{M}, (u, \mathsf{u}) \models \varphi$ and $M, u \models \chi_{\mathsf{u}}$, as wanted.

Soundness of AMC

Proof of Action and Common Knowledge Axiom (3/3)

- $\mathbf{n = k + 1}$ Let $(u, \mathsf{u}) \sim^{k+1}_{\otimes\mathsf{M};E_B} (s, \mathsf{s})$.
  Equivalently, there is $(t, \mathsf{t}) \in S_{\otimes\mathsf{M}}$ s.t. $(s, \mathsf{s}) \sim^{k}_{\otimes\mathsf{M};E_B} (t, \mathsf{t}) \sim_{\otimes\mathsf{M};a} (u, \mathsf{u})$, for some $a \in B$.

  $M, t \models \mathsf{pre}(\mathsf{t})$ $(t, \mathsf{t}) \in S_{\otimes\mathsf{M}}$
  $M, t \models \chi_{\mathsf{t}}$ by I.H.

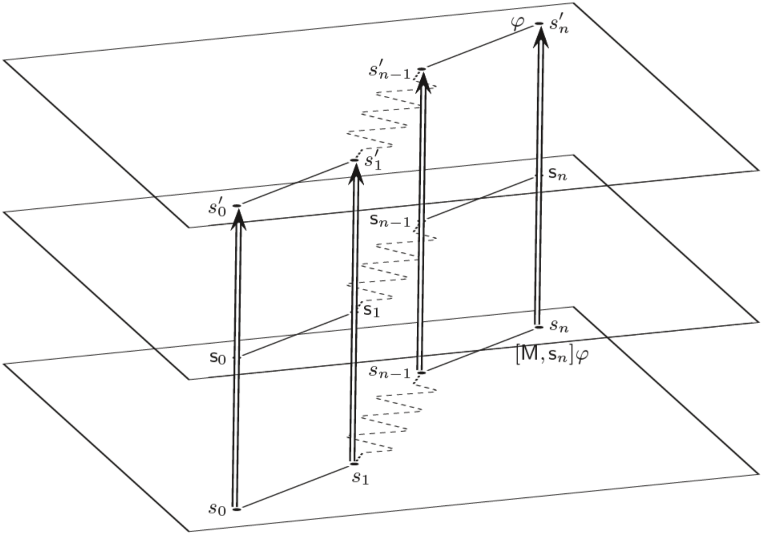
  Thus by $\models \chi_{\mathsf{t}} \wedge \mathsf{pre}(\mathsf{t}) \rightarrow K_a\chi_{\mathsf{u}}$, we have $M, t \models K_a\chi_{\mathsf{u}}$ and as $u \sim_a t$ we have $M, u \models \chi_{\mathsf{u}}$.

  $M, u \models \mathsf{pre}(\mathsf{u})$ $(u, \mathsf{u}) \in S_{\otimes\mathsf{M}}$

  Thus by $M, u \models \chi_{\mathsf{u}}$ and $\models \chi_{\mathsf{u}} \rightarrow [\mathsf{M}, \mathsf{u}]\varphi$ we have $M \otimes \mathsf{M}, (u, \mathsf{u}) \models \varphi$ and $M, u \models \chi_{\mathsf{u}}$, as wanted.
  By induction principle we get the required statement.

$\square$

Action and common knowledge



Example 6.38

$$\mathbf{AMC} \vdash [\mathsf{Read}, \mathsf{p}]\, K_a p$$

Example 6.38

$$\mathbf{AMC} \vdash [\mathsf{Read}, \mathsf{p}]\, K_a p$$

$\varphi_1 : p \to p$ tautology

Example 6.38

$$\mathbf{AMC} \vdash [\mathsf{Read}, \mathsf{p}]\, K_a p$$

$\varphi_1 : p \to p$ — tautology

$\varphi_2 : [\mathsf{Read}, \mathsf{p}]\, p \leftrightarrow (p \to p)$ — $\mathsf{pre}(\mathsf{p}) = p$, atomic permanence

Example 6.38

$$\mathbf{AMC} \vdash [\mathsf{Read}, \mathsf{p}]\, K_a p$$

φ_1	$: p \rightarrow p$	tautology
φ_2	$: [\mathsf{Read}, \mathsf{p}]\, p \leftrightarrow (p \rightarrow p)$	$\mathsf{pre}(\mathsf{p}) = p$, atomic permanence
φ_3	$: [\mathsf{Read}, \mathsf{p}]\, p$	1,2, Pr.

Example 6.38

$$\mathbf{AMC} \vdash [\mathsf{Read}, \mathsf{p}]\, K_a p$$

$\varphi_1 : p \rightarrow p$	tautology
$\varphi_2 : [\mathsf{Read}, \mathsf{p}]\, p \leftrightarrow (p \rightarrow p)$	$\mathsf{pre}(\mathsf{p}) = p$, atomic permanence
$\varphi_3 : [\mathsf{Read}, \mathsf{p}]\, p$	1,2, Pr.
$\varphi_4 : K_a\, [\mathsf{Read}, \mathsf{p}]\, p$	3, Nec of K_a

Example 6.38

$$\mathbf{AMC} \vdash [\mathsf{Read}, \mathsf{p}]\, K_a p$$

$\varphi_1 : p \to p$	tautology
$\varphi_2 : [\mathsf{Read}, \mathsf{p}]\, p \leftrightarrow (p \to p)$	$\mathsf{pre}(\mathsf{p}) = p$, atomic permanence
$\varphi_3 : [\mathsf{Read}, \mathsf{p}]\, p$	1,2, Pr.
$\varphi_4 : K_a\, [\mathsf{Read}, \mathsf{p}]\, p$	3, Nec of K_a
$\varphi_5 : p \to K_a\, [\mathsf{Read}, \mathsf{p}]\, p$	4, weakening

Example 6.38

$$\mathbf{AMC} \vdash [\mathsf{Read}, \mathsf{p}]\, K_a p$$

$\varphi_1 : p \to p$	tautology
$\varphi_2 : [\mathsf{Read}, \mathsf{p}]\, p \leftrightarrow (p \to p)$	$\mathsf{pre}(\mathsf{p}) = p$, atomic permanence
$\varphi_3 : [\mathsf{Read}, \mathsf{p}]\, p$	1,2, Pr.
$\varphi_4 : K_a\, [\mathsf{Read}, \mathsf{p}]\, p$	3, Nec of K_a
$\varphi_5 : p \to K_a\, [\mathsf{Read}, \mathsf{p}]\, p$	4, weakening
$\varphi_6 : [\mathsf{Read}, \mathsf{p}]\, K_a p \leftrightarrow \left(p \to \bigwedge_{\mathsf{p} \sim_a \mathsf{s}} K_a\, [\mathsf{Read}, \mathsf{s}]\, p\right)$	$[\mathsf{p}]_{\sim_a} = \{p\}$, act&kn

Example 6.38

$$\mathbf{AMC} \vdash [\mathsf{Read}, \mathsf{p}]\, K_a p$$

$\varphi_1 : p \to p$	tautology
$\varphi_2 : [\mathsf{Read}, \mathsf{p}]\, p \leftrightarrow (p \to p)$	$\mathsf{pre}(\mathsf{p}) = p$, atomic permanence
$\varphi_3 : [\mathsf{Read}, \mathsf{p}]\, p$	1,2, Pr.
$\varphi_4 : K_a\, [\mathsf{Read}, \mathsf{p}]\, p$	3, Nec of K_a
$\varphi_5 : p \to K_a\, [\mathsf{Read}, \mathsf{p}]\, p$	4, weakening
$\varphi_6 : [\mathsf{Read}, \mathsf{p}]\, K_a p \leftrightarrow \left(p \to \bigwedge_{\mathsf{p} \sim_a \mathsf{s}} K_a\, [\mathsf{Read}, \mathsf{s}]\, p\right)$	$[\mathsf{p}]_{\sim_a} = \{p\}$, act&kn
$\varphi_7 : [\mathsf{Read}, \mathsf{p}]\, K_a p$	5,6, Pr.

Is $[\alpha]$ a normal modal operator?

The necessitation rule holds for $[\alpha]$. Does the axiom **K**, i.e.

$$[\alpha]\,(\varphi \to \psi) \to [\alpha]\,\varphi \to [\alpha]\,\psi$$

also hold?

Is $[\alpha]$ a normal modal operator?

The necessitation rule holds for $[\alpha]$. Does the axiom **K**, i.e.

$$[\alpha]\,(\varphi \to \psi) \to [\alpha]\,\varphi \to [\alpha]\,\psi$$

also hold?

YES!

$[\alpha]$ respects axiom **K**. *Proof* (1/5)

By action composition, non-deterministic choice and

$$(a \to b \to c) \wedge (a' \to b' \to c') \to a \wedge a' \to b \wedge b' \to c \wedge c'$$

axioms, we get that it suffices to show that, for any pointed action model (M, s)

$$\mathbf{AMC} \vdash [\mathsf{M}, \mathsf{s}]\,(\varphi \to \psi) \to [\mathsf{M}, \mathsf{s}]\,\varphi \to [\mathsf{M}, \mathsf{s}]\,\psi$$

$[\alpha]$ respects axiom **K**. *Proof* (1/5)

By action composition, non-deterministic choice and

$$(a \to b \to c) \wedge (a' \to b' \to c') \to a \wedge a' \to b \wedge b' \to c \wedge c'$$

axioms, we get that it suffices to show that, for any pointed action model (M, s)

$$\mathbf{AMC} \vdash [\mathsf{M}, \mathsf{s}]\,(\varphi \to \psi) \to [\mathsf{M}, \mathsf{s}]\,\varphi \to [\mathsf{M}, \mathsf{s}]\,\psi$$

Note that $[\mathsf{M}, \mathsf{s}]\,(\varphi \to \psi)$ is an abbreviation for $[\mathsf{M}, \mathsf{s}]\,\neg\,(\varphi \wedge \neg\psi)$

$[\alpha]$ respects axiom **K**. *Proof* (2/5)

$$[\mathsf{M},\mathsf{s}]\,\neg\,(\varphi \wedge \neg\psi) \to \mathsf{pre}(\mathsf{s}) \to \neg\,[\mathsf{M},\mathsf{s}]\,(\varphi \wedge \neg\psi) \qquad \text{act.\&neg., Pr.}$$

$[\alpha]$ respects axiom **K**. *Proof* (2/5)

$$[\mathsf{M}, \mathsf{s}] \neg (\varphi \wedge \neg\psi) \rightarrow \mathsf{pre}(\mathsf{s}) \rightarrow \neg [\mathsf{M}, \mathsf{s}] (\varphi \wedge \neg\psi) \qquad \text{act.\&neg., Pr.}$$

$$[\mathsf{M}, \mathsf{s}] \neg (\varphi \wedge \neg\psi) \rightarrow \mathsf{pre}(\mathsf{s}) \rightarrow \neg ([\mathsf{M}, \mathsf{s}] \varphi \wedge [\mathsf{M}, \mathsf{s}] \neg\psi) \qquad \text{act.\&conj., Pr.}$$

$[\alpha]$ respects axiom **K**. *Proof* (2/5)

$$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\rightarrow\mathsf{pre(s)}\rightarrow\neg[\mathsf{M},\mathsf{s}](\varphi\wedge\neg\psi) \qquad \text{act.\&neg., Pr.}$$

$$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\rightarrow\mathsf{pre(s)}\rightarrow\neg([\mathsf{M},\mathsf{s}]\varphi\wedge[\mathsf{M},\mathsf{s}]\neg\psi) \qquad \text{act.\&conj., Pr.}$$

$$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\rightarrow\mathsf{pre(s)}\rightarrow[\mathsf{M},\mathsf{s}]\varphi\rightarrow\neg[\mathsf{M},\mathsf{s}]\neg\psi \qquad \text{Pr.}$$

$[\alpha]$ respects axiom **K**. *Proof* (2/5)

$$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\to\mathsf{pre}(\mathsf{s})\to\neg[\mathsf{M},\mathsf{s}](\varphi\wedge\neg\psi) \qquad \text{act.\&neg., Pr.}$$

$$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\to\mathsf{pre}(\mathsf{s})\to\neg([\mathsf{M},\mathsf{s}]\varphi\wedge[\mathsf{M},\mathsf{s}]\neg\psi) \qquad \text{act.\&conj., Pr.}$$

$$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\to\mathsf{pre}(\mathsf{s})\to[\mathsf{M},\mathsf{s}]\varphi\to\neg[\mathsf{M},\mathsf{s}]\neg\psi \qquad \text{Pr.}$$

$$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\to[\mathsf{M},\mathsf{s}]\varphi\to\mathsf{pre}(\mathsf{s})\to\neg(\mathsf{pre}(\mathsf{s})\to\neg[\mathsf{M},\mathsf{s}]\psi) \qquad \text{Pr., act.\&neg.}$$

$[\alpha]$ respects axiom **K**. *Proof* (2/5)

$$[\mathsf{M},\mathsf{s}]\,\neg\,(\varphi \wedge \neg\psi) \to \mathsf{pre}(\mathsf{s}) \to \neg\,[\mathsf{M},\mathsf{s}]\,(\varphi \wedge \neg\psi) \qquad \text{act.\&neg., Pr.}$$

$$[\mathsf{M},\mathsf{s}]\,\neg\,(\varphi \wedge \neg\psi) \to \mathsf{pre}(\mathsf{s}) \to \neg\,([\mathsf{M},\mathsf{s}]\,\varphi \wedge [\mathsf{M},\mathsf{s}]\,\neg\psi) \qquad \text{act.\&conj., Pr.}$$

$$[\mathsf{M},\mathsf{s}]\,\neg\,(\varphi \wedge \neg\psi) \to \mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,\varphi \to \neg\,[\mathsf{M},\mathsf{s}]\,\neg\psi \qquad \text{Pr.}$$

$$[\mathsf{M},\mathsf{s}]\,\neg\,(\varphi \wedge \neg\psi) \to [\mathsf{M},\mathsf{s}]\,\varphi \to \mathsf{pre}(\mathsf{s}) \to \neg\,(\mathsf{pre}(\mathsf{s}) \to \neg\,[\mathsf{M},\mathsf{s}]\,\psi) \qquad \text{Pr., act.\&neg.}$$

$$[\mathsf{M},\mathsf{s}]\,\neg\,(\varphi \wedge \neg\psi) \to [\mathsf{M},\mathsf{s}]\,\varphi \to \mathsf{pre}(\mathsf{s}) \to \mathsf{pre}(\mathsf{s}) \wedge [\mathsf{M},\mathsf{s}]\,\psi \qquad \text{Pr.}$$

$[\alpha]$ respects axiom **K**. *Proof* (2/5)

$$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\rightarrow\mathsf{pre}(\mathsf{s})\rightarrow\neg[\mathsf{M},\mathsf{s}](\varphi\wedge\neg\psi) \qquad \text{act.\&neg., Pr.}$$

$$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\rightarrow\mathsf{pre}(\mathsf{s})\rightarrow\neg([\mathsf{M},\mathsf{s}]\varphi\wedge[\mathsf{M},\mathsf{s}]\neg\psi) \qquad \text{act.\&conj., Pr.}$$

$$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\rightarrow\mathsf{pre}(\mathsf{s})\rightarrow[\mathsf{M},\mathsf{s}]\varphi\rightarrow\neg[\mathsf{M},\mathsf{s}]\neg\psi \qquad \text{Pr.}$$

$$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\rightarrow[\mathsf{M},\mathsf{s}]\varphi\rightarrow\mathsf{pre}(\mathsf{s})\rightarrow\neg(\mathsf{pre}(\mathsf{s})\rightarrow\neg[\mathsf{M},\mathsf{s}]\psi) \qquad \text{Pr., act.\&neg.}$$

$$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\rightarrow[\mathsf{M},\mathsf{s}]\varphi\rightarrow\mathsf{pre}(\mathsf{s})\rightarrow\mathsf{pre}(\mathsf{s})\wedge[\mathsf{M},\mathsf{s}]\psi \qquad \text{Pr.}$$

$$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\rightarrow[\mathsf{M},\mathsf{s}]\varphi\rightarrow\mathsf{pre}(\mathsf{s})\rightarrow[\mathsf{M},\mathsf{s}]\psi \qquad \text{Pr.}$$

$[\alpha]$ respects axiom **K**. *Proof* (2/5)

$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\to\mathsf{pre}(\mathsf{s})\to\neg[\mathsf{M},\mathsf{s}](\varphi\wedge\neg\psi)$	act.&neg., Pr.
$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\to\mathsf{pre}(\mathsf{s})\to\neg([\mathsf{M},\mathsf{s}]\varphi\wedge[\mathsf{M},\mathsf{s}]\neg\psi)$	act.&conj., Pr.
$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\to\mathsf{pre}(\mathsf{s})\to[\mathsf{M},\mathsf{s}]\varphi\to\neg[\mathsf{M},\mathsf{s}]\neg\psi$	Pr.
$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\to[\mathsf{M},\mathsf{s}]\varphi\to\mathsf{pre}(\mathsf{s})\to\neg(\mathsf{pre}(\mathsf{s})\to\neg[\mathsf{M},\mathsf{s}]\psi)$	Pr., act.&neg.
$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\to[\mathsf{M},\mathsf{s}]\varphi\to\mathsf{pre}(\mathsf{s})\to\mathsf{pre}(\mathsf{s})\wedge[\mathsf{M},\mathsf{s}]\psi$	Pr.
$[\mathsf{M},\mathsf{s}]\neg(\varphi\wedge\neg\psi)\to[\mathsf{M},\mathsf{s}]\varphi\to\mathsf{pre}(\mathsf{s})\to[\mathsf{M},\mathsf{s}]\psi$	Pr.

Thus it suffices to show that for any ψ

$$\mathbf{AMC}\vdash(\mathsf{pre}(\mathsf{s})\to[\mathsf{M},\mathsf{s}]\psi)\to[\mathsf{M},\mathsf{s}]\psi \qquad (*)$$

$[\alpha]$ respects axiom **K**. *Proof* (3/5)

$$\mathbf{AMC} \vdash (\mathsf{pre}(\mathsf{s}) \rightarrow [\mathsf{M}, \mathsf{s}]\,\psi) \rightarrow [\mathsf{M}, \mathsf{s}]\,\psi$$

We will prove it by induction on the complexity of ψ.

- $\psi := p$

$[\alpha]$ respects axiom **K**. *Proof* (3/5)

$$\mathbf{AMC} \vdash (\mathsf{pre(s)} \to [\mathsf{M,s}]\,\psi) \to [\mathsf{M,s}]\,\psi$$

We will prove it by induction on the complexity of ψ.

- $\psi := p$

  $(\mathsf{pre(s)} \to [\mathsf{M,s}]\,p) \to \mathsf{pre(s)} \to \mathsf{pre(s)} \to p$ at. perm., Pr.

$[\alpha]$ respects axiom **K**. *Proof* (3/5)

$$\mathbf{AMC} \vdash (\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,\psi) \to [\mathsf{M},\mathsf{s}]\,\psi$$

We will prove it by induction on the complexity of ψ.

- $\psi := p$

  $(\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,p) \to \mathsf{pre}(\mathsf{s}) \to \mathsf{pre}(\mathsf{s}) \to p$ at. perm., Pr.

  $(\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,p) \to \mathsf{pre}(\mathsf{s}) \to p$ Pr.

$[\alpha]$ respects axiom **K**. *Proof* (3/5)

$$\mathbf{AMC} \vdash (\mathsf{pre}(\mathsf{s}) \rightarrow [\mathsf{M},\mathsf{s}]\,\psi) \rightarrow [\mathsf{M},\mathsf{s}]\,\psi$$

We will prove it by induction on the complexity of ψ.

- $\psi := p$

  $(\mathsf{pre}(\mathsf{s}) \rightarrow [\mathsf{M},\mathsf{s}]\,p) \rightarrow \mathsf{pre}(\mathsf{s}) \rightarrow \mathsf{pre}(\mathsf{s}) \rightarrow p$ — at. perm., Pr.

  $(\mathsf{pre}(\mathsf{s}) \rightarrow [\mathsf{M},\mathsf{s}]\,p) \rightarrow \mathsf{pre}(\mathsf{s}) \rightarrow p$ — Pr.

  $(\mathsf{pre}(\mathsf{s}) \rightarrow [\mathsf{M},\mathsf{s}]\,p) \rightarrow [\mathsf{M},\mathsf{s}]\,p$ — at. perm., Pr.

$[\alpha]$ respects axiom **K**. *Proof* (3/5)

$$\textbf{AMC} \vdash (\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,\psi) \to [\mathsf{M},\mathsf{s}]\,\psi$$

We will prove it by induction on the complexity of ψ.

- $\psi := p$

  $(\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,p) \to \mathsf{pre}(\mathsf{s}) \to \mathsf{pre}(\mathsf{s}) \to p$ at. perm., Pr.

  $(\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,p) \to \mathsf{pre}(\mathsf{s}) \to p$ Pr.

  $(\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,p) \to [\mathsf{M},\mathsf{s}]\,p$ at. perm., Pr.

- $\psi := \neg\psi$ or $\psi := K_a\psi$ similarly with previous, without using induction.

$[\alpha]$ respects axiom **K**. *Proof* (3/5)

$$\mathbf{AMC} \vdash (\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,\psi) \to [\mathsf{M},\mathsf{s}]\,\psi$$

We will prove it by induction on the complexity of ψ.

- $\psi := p$

  $(\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,p) \to \mathsf{pre}(\mathsf{s}) \to \mathsf{pre}(\mathsf{s}) \to p$ at. perm., Pr.

  $(\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,p) \to \mathsf{pre}(\mathsf{s}) \to p$ Pr.

  $(\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,p) \to [\mathsf{M},\mathsf{s}]\,p$ at. perm., Pr.

- $\psi := \neg\psi$ or $\psi := K_a\psi$ similarly with previous, without using induction.

- $\psi := \chi \wedge \psi$ EZ, by using induction.

$[\alpha]$ respects axiom **K**. *Proof* (3/5)

$$\textbf{AMC} \vdash (\mathsf{pre}(\mathsf{s}) \to [\mathsf{M}, \mathsf{s}]\,\psi) \to [\mathsf{M}, \mathsf{s}]\,\psi$$

We will prove it by induction on the complexity of ψ.

- $\psi := p$

  $(\mathsf{pre}(\mathsf{s}) \to [\mathsf{M}, \mathsf{s}]\,p) \to \mathsf{pre}(\mathsf{s}) \to \mathsf{pre}(\mathsf{s}) \to p$ at. perm., Pr.

  $(\mathsf{pre}(\mathsf{s}) \to [\mathsf{M}, \mathsf{s}]\,p) \to \mathsf{pre}(\mathsf{s}) \to p$ Pr.

  $(\mathsf{pre}(\mathsf{s}) \to [\mathsf{M}, \mathsf{s}]\,p) \to [\mathsf{M}, \mathsf{s}]\,p$ at. perm., Pr.

- $\psi := \neg\psi$ or $\psi := K_a\psi$ similarly with previous, without using induction.

- $\psi := \chi \wedge \psi$ EZ, by using induction.

- $\psi := [\alpha]\,\psi$ W.l.o.g. $\alpha := (\mathsf{M}', \mathsf{s}')$,

$[\alpha]$ respects axiom **K**. *Proof* (3/5)

$$\textbf{AMC} \vdash (\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,\psi) \to [\mathsf{M},\mathsf{s}]\,\psi$$

We will prove it by induction on the complexity of ψ.

- $\psi := p$

  $(\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,p) \to \mathsf{pre}(\mathsf{s}) \to \mathsf{pre}(\mathsf{s}) \to p$ at. perm., Pr.

  $(\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,p) \to \mathsf{pre}(\mathsf{s}) \to p$ Pr.

  $(\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,p) \to [\mathsf{M},\mathsf{s}]\,p$ at. perm., Pr.

- $\psi := \neg\psi$ or $\psi := K_a\psi$ similarly with previous, without using induction.

- $\psi := \chi \wedge \psi$ EZ, by using induction.

- $\psi := [\alpha]\,\psi$ W.l.o.g. $\alpha := (\mathsf{M}', \mathsf{s}')$,
  Hint: $\mathsf{pre}((\mathsf{s},\mathsf{s}')) = \mathsf{pre}(\mathsf{s}) \wedge \mathsf{pre}(\mathsf{s}')$, by using induction.

$[\alpha]$ respects axiom **K**. *Proof* (4/5)

$$\mathbf{AMC} \vdash (\mathsf{pre}(\mathsf{s}) \rightarrow [\mathsf{M}, \mathsf{s}]\,\psi) \rightarrow [\mathsf{M}, \mathsf{s}]\,\psi$$

- $\psi := C_B\psi$ We denote

$$\chi_{\mathsf{t}} := \mathsf{pre}(\mathsf{t}) \rightarrow [\mathsf{M}, \mathsf{t}]\, C_B\psi,$$

for any $\mathsf{t} \sim_B \mathsf{s}$.

$[\alpha]$ respects axiom **K**. *Proof* (4/5)

$$\mathbf{AMC} \vdash (\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,\psi) \to [\mathsf{M},\mathsf{s}]\,\psi$$

- $\psi := C_B\psi$ We denote

  $$\chi_\mathsf{t} := \mathsf{pre}(\mathsf{t}) \to [\mathsf{M},\mathsf{t}]\,C_B\psi,$$

  for any $\mathsf{t} \sim_B \mathsf{s}$.
  We want to show that for any $\mathsf{t} \sim_B \mathsf{s}$ and for any $a \in B$ and $\mathsf{u} \sim_a \mathsf{t}$

  $$\mathbf{AMC} \vdash \chi_\mathsf{t} \to [\mathsf{M},\mathsf{t}]\,\psi$$

  and

  $$\mathbf{AMC} \vdash \chi_\mathsf{t} \wedge \mathsf{pre}(\mathsf{t}) \to [\mathsf{M},\mathsf{t}]\,K_a\chi_\mathsf{u}$$

$[\alpha]$ respects axiom **K**. *Proof* (4/5)

$$\mathbf{AMC} \vdash (\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,\psi) \to [\mathsf{M},\mathsf{s}]\,\psi$$

- $\psi := C_B\psi$ We denote

  $$\chi_\mathsf{t} := \mathsf{pre}(\mathsf{t}) \to [\mathsf{M},\mathsf{t}]\,C_B\psi,$$

  for any $\mathsf{t} \sim_B \mathsf{s}$.
  We want to show that for any $\mathsf{t} \sim_B \mathsf{s}$ and for any $a \in B$ and $\mathsf{u} \sim_a \mathsf{t}$

  $$\mathbf{AMC} \vdash \chi_\mathsf{t} \to [\mathsf{M},\mathsf{t}]\,\psi$$

  and

  $$\mathbf{AMC} \vdash \chi_\mathsf{t} \wedge \mathsf{pre}(\mathsf{t}) \to [\mathsf{M},\mathsf{t}]\,K_a\chi_\mathsf{u}$$

  Then by action and comm. knowl. axiom we have

  $$\mathbf{AMC} \vdash (\mathsf{pre}(\mathsf{s}) \to [\mathsf{M},\mathsf{s}]\,C_B\psi) \to [\mathsf{M},\mathsf{s}]\,C_B\psi$$

$[\alpha]$ respects axiom **K**. *Proof* (5/5)

$$\mathbf{AMC} \vdash \chi_{\mathsf{t}} \to [\mathsf{M}, \mathsf{t}]\, \psi$$

$[\alpha]$ respects axiom **K**. *Proof* (5/5)

$$\mathbf{AMC} \vdash \chi_{\mathsf{t}} \to [\mathsf{M}, \mathsf{t}]\,\psi$$

$C_B\psi \to \psi$ mix, Pr.

$[\alpha]$ respects axiom **K**. *Proof* (5/5)

$$\mathbf{AMC} \vdash \chi_t \to [\mathsf{M}, \mathsf{t}]\, \psi$$

$C_B\psi \to \psi$	mix, Pr.
$[\mathsf{M}, \mathsf{t}]\, C_B\psi \to [\mathsf{M}, \mathsf{t}]\, \psi$	I.H. for **K**, $\psi \prec C_B\psi$

$[\alpha]$ respects axiom **K**. *Proof* (5/5)

$$\mathbf{AMC} \vdash \chi_{\mathsf{t}} \to [\mathsf{M},\mathsf{t}]\,\psi$$

$C_B\psi \to \psi$	mix, Pr.
$[\mathsf{M},\mathsf{t}]\,C_B\psi \to [\mathsf{M},\mathsf{t}]\,\psi$	I.H. for **K**, $\psi \prec C_B\psi$
$(\mathsf{pre}(\mathsf{t}) \to [\mathsf{M},\mathsf{t}]\,C_B\psi) \to \mathsf{pre}(\mathsf{t}) \to [\mathsf{M},\mathsf{t}]\,\psi$	Pr.

$[\alpha]$ respects axiom **K**. *Proof* (5/5)

$$\mathbf{AMC} \vdash \chi_{\mathsf{t}} \to [\mathsf{M},\mathsf{t}]\,\psi$$

$C_B\psi \to \psi$	mix, Pr.
$[\mathsf{M},\mathsf{t}]\, C_B\psi \to [\mathsf{M},\mathsf{t}]\,\psi$	I.H. for **K**, $\psi \prec C_B\psi$
$(\mathsf{pre}(\mathsf{t}) \to [\mathsf{M},\mathsf{t}]\, C_B\psi) \to \mathsf{pre}(\mathsf{t}) \to [\mathsf{M},\mathsf{t}]\,\psi$	Pr.
$\chi_{\mathsf{t}} \to [\mathsf{M},\mathsf{t}]\,\psi$	I.H. for (*), $\psi \prec C_B\psi$

$[\alpha]$ respects axiom **K**. *Proof* (5/5)

$$\mathbf{AMC} \vdash \chi_{\mathsf{t}} \rightarrow [\mathsf{M}, \mathsf{t}]\, \psi$$

$C_B\psi \rightarrow \psi$	mix, Pr.
$[\mathsf{M}, \mathsf{t}]\, C_B\psi \rightarrow [\mathsf{M}, \mathsf{t}]\, \psi$	I.H. for **K**, $\psi \prec C_B\psi$
$(\mathsf{pre}(\mathsf{t}) \rightarrow [\mathsf{M}, \mathsf{t}]\, C_B\psi) \rightarrow \mathsf{pre}(\mathsf{t}) \rightarrow [\mathsf{M}, \mathsf{t}]\, \psi$	Pr.
$\chi_{\mathsf{t}} \rightarrow [\mathsf{M}, \mathsf{t}]\, \psi$	I.H. for (*), $\psi \prec C_B\psi$

$$\mathbf{AMC} \vdash \chi_{\mathsf{t}} \wedge \mathsf{pre}(\mathsf{t}) \rightarrow [\mathsf{M}, \mathsf{t}]\, K_a\chi_{\mathsf{u}}$$

$C_B\psi \rightarrow K_a C_B\psi$	mix, Pr.

$[\alpha]$ respects axiom **K**. *Proof* (5/5)

$$\mathbf{AMC} \vdash \chi_{\mathsf{t}} \to [\mathsf{M},\mathsf{t}]\,\psi$$

$C_B\psi \to \psi$	mix, Pr.
$[\mathsf{M},\mathsf{t}]\,C_B\psi \to [\mathsf{M},\mathsf{t}]\,\psi$	I.H. for **K**, $\psi \prec C_B\psi$
$(\mathsf{pre}(\mathsf{t}) \to [\mathsf{M},\mathsf{t}]\,C_B\psi) \to \mathsf{pre}(\mathsf{t}) \to [\mathsf{M},\mathsf{t}]\,\psi$	Pr.
$\chi_{\mathsf{t}} \to [\mathsf{M},\mathsf{t}]\,\psi$	I.H. for (*), $\psi \prec C_B\psi$

$$\mathbf{AMC} \vdash \chi_{\mathsf{t}} \land \mathsf{pre}(\mathsf{t}) \to [\mathsf{M},\mathsf{t}]\,K_a\chi_{\mathsf{u}}$$

$C_B\psi \to K_a C_B\psi$	mix, Pr.
$[\mathsf{M},\mathsf{t}]\,C_B\psi \to [\mathsf{M},\mathsf{t}]\,K_a C_B\psi$	**K** holds w/o I.H. for K_a

$[\alpha]$ respects axiom K. *Proof* (5/5)

$$\mathbf{AMC} \vdash \chi_{\mathsf{t}} \to [\mathsf{M}, \mathsf{t}]\, \psi$$

$C_B\psi \to \psi$	mix, Pr.
$[\mathsf{M}, \mathsf{t}]\, C_B\psi \to [\mathsf{M}, \mathsf{t}]\, \psi$	I.H. for **K**, $\psi \prec C_B\psi$
$(\mathsf{pre}(\mathsf{t}) \to [\mathsf{M}, \mathsf{t}]\, C_B\psi) \to \mathsf{pre}(\mathsf{t}) \to [\mathsf{M}, \mathsf{t}]\, \psi$	Pr.
$\chi_{\mathsf{t}} \to [\mathsf{M}, \mathsf{t}]\, \psi$	I.H. for (*), $\psi \prec C_B\psi$

$$\mathbf{AMC} \vdash \chi_{\mathsf{t}} \wedge \mathsf{pre}(\mathsf{t}) \to [\mathsf{M}, \mathsf{t}]\, K_a\chi_{\mathsf{u}}$$

$C_B\psi \to K_a C_B\psi$	mix, Pr.
$[\mathsf{M}, \mathsf{t}]\, C_B\psi \to [\mathsf{M}, \mathsf{t}]\, K_a C_B\psi$	**K** holds w/o I.H. for K_a
$[\mathsf{M}, \mathsf{t}]\, C_B\psi \to \mathsf{pre}(\mathsf{t}) \to \bigwedge_{\mathsf{u} \sim_a \mathsf{t}} K_a\, [\mathsf{M}, \mathsf{u}]\, C_B\psi$	act.&kn., Pr.

$[\alpha]$ respects axiom **K**. *Proof* (5/5)

$$\mathbf{AMC} \vdash \chi_t \to [\mathsf{M}, \mathsf{t}]\, \psi$$

$C_B\psi \to \psi$	mix, Pr.
$[\mathsf{M}, \mathsf{t}]\, C_B\psi \to [\mathsf{M}, \mathsf{t}]\, \psi$	I.H. for **K**, $\psi \prec C_B\psi$
$(\mathsf{pre}(\mathsf{t}) \to [\mathsf{M}, \mathsf{t}]\, C_B\psi) \to \mathsf{pre}(\mathsf{t}) \to [\mathsf{M}, \mathsf{t}]\, \psi$	Pr.
$\chi_t \to [\mathsf{M}, \mathsf{t}]\, \psi$	I.H. for (*), $\psi \prec C_B\psi$

$$\mathbf{AMC} \vdash \chi_t \wedge \mathsf{pre}(\mathsf{t}) \to [\mathsf{M}, \mathsf{t}]\, K_a \chi_u$$

$C_B\psi \to K_a C_B\psi$	mix, Pr.
$[\mathsf{M}, \mathsf{t}]\, C_B\psi \to [\mathsf{M}, \mathsf{t}]\, K_a C_B\psi$	**K** holds w/o I.H. for K_a
$[\mathsf{M}, \mathsf{t}]\, C_B\psi \to \mathsf{pre}(\mathsf{t}) \to \bigwedge_{\mathsf{u} \sim_a \mathsf{t}} K_a\, [\mathsf{M}, \mathsf{u}]\, C_B\psi$	act.&kn., Pr.
$[\mathsf{M}, \mathsf{t}]\, C_B\psi \to \mathsf{pre}(\mathsf{t}) \to K_a \chi_u$	$(x \to K_a y) \to x \to K_a (z \to y)$

$[\alpha]$ respects axiom **K**. *Proof* (5/5)

$$\mathbf{AMC} \vdash \chi_{\mathsf{t}} \rightarrow [\mathsf{M},\mathsf{t}]\,\psi$$

$$\begin{array}{ll}
C_B\psi \rightarrow \psi & \text{mix, Pr.} \\
[\mathsf{M},\mathsf{t}]\,C_B\psi \rightarrow [\mathsf{M},\mathsf{t}]\,\psi & \text{I.H. for } \mathbf{K},\ \psi \prec C_B\psi \\
(\mathsf{pre}(\mathsf{t}) \rightarrow [\mathsf{M},\mathsf{t}]\,C_B\psi) \rightarrow \mathsf{pre}(\mathsf{t}) \rightarrow [\mathsf{M},\mathsf{t}]\,\psi & \text{Pr.} \\
\chi_{\mathsf{t}} \rightarrow [\mathsf{M},\mathsf{t}]\,\psi & \text{I.H. for (*)},\ \psi \prec C_B\psi
\end{array}$$

$$\mathbf{AMC} \vdash \chi_{\mathsf{t}} \wedge \mathsf{pre}(\mathsf{t}) \rightarrow [\mathsf{M},\mathsf{t}]\,K_a\chi_{\mathsf{u}}$$

$$\begin{array}{ll}
C_B\psi \rightarrow K_aC_B\psi & \text{mix, Pr.} \\
[\mathsf{M},\mathsf{t}]\,C_B\psi \rightarrow [\mathsf{M},\mathsf{t}]\,K_aC_B\psi & \mathbf{K} \text{ holds w/o I.H. for } K_a \\
[\mathsf{M},\mathsf{t}]\,C_B\psi \rightarrow \mathsf{pre}(\mathsf{t}) \rightarrow \bigwedge_{\mathsf{u}\sim_a\mathsf{t}} K_a\,[\mathsf{M},\mathsf{u}]\,C_B\psi & \text{act.\&kn., Pr.} \\
[\mathsf{M},\mathsf{t}]\,C_B\psi \rightarrow \mathsf{pre}(\mathsf{t}) \rightarrow K_a\chi_{\mathsf{u}} & (x \rightarrow K_ay) \rightarrow x \rightarrow K_a\,(z \rightarrow y) \\
[\mathsf{M},\mathsf{t}]\,C_B\psi \wedge \mathsf{pre}(\mathsf{t}) \rightarrow K_a\chi_{\mathsf{u}} & \text{Pr.}
\end{array}$$

$[\alpha]$ respects axiom **K**. *Proof* (5/5)

$$\mathbf{AMC} \vdash \chi_{\mathsf{t}} \to [\mathsf{M}, \mathsf{t}]\, \psi$$

$C_B\psi \to \psi$	mix, Pr.
$[\mathsf{M}, \mathsf{t}]\, C_B\psi \to [\mathsf{M}, \mathsf{t}]\, \psi$	I.H. for **K**, $\psi \prec C_B\psi$
$(\mathsf{pre}(\mathsf{t}) \to [\mathsf{M}, \mathsf{t}]\, C_B\psi) \to \mathsf{pre}(\mathsf{t}) \to [\mathsf{M}, \mathsf{t}]\, \psi$	Pr.
$\chi_{\mathsf{t}} \to [\mathsf{M}, \mathsf{t}]\, \psi$	I.H. for (*), $\psi \prec C_B\psi$

$$\mathbf{AMC} \vdash \chi_{\mathsf{t}} \wedge \mathsf{pre}(\mathsf{t}) \to [\mathsf{M}, \mathsf{t}]\, K_a\chi_{\mathsf{u}}$$

$C_B\psi \to K_a C_B\psi$	mix, Pr.
$[\mathsf{M}, \mathsf{t}]\, C_B\psi \to [\mathsf{M}, \mathsf{t}]\, K_a C_B\psi$	**K** holds w/o I.H. for K_a
$[\mathsf{M}, \mathsf{t}]\, C_B\psi \to \mathsf{pre}(\mathsf{t}) \to \bigwedge_{\mathsf{u} \sim_a \mathsf{t}} K_a\, [\mathsf{M}, \mathsf{u}]\, C_B\psi$	act.&kn., Pr.
$[\mathsf{M}, \mathsf{t}]\, C_B\psi \to \mathsf{pre}(\mathsf{t}) \to K_a\chi_{\mathsf{u}}$	$(x \to K_a y) \to x \to K_a\,(z \to y)$
$[\mathsf{M}, \mathsf{t}]\, C_B\psi \wedge \mathsf{pre}(\mathsf{t}) \to K_a\chi_{\mathsf{u}}$	Pr.
$\chi_{\mathsf{t}} \wedge \mathsf{pre}(\mathsf{t}) \to K_a\chi_{\mathsf{u}}$	$(x \wedge y \to z) \to (y \to x) \wedge y \to z$

$\square$

- DEMO stands for Dynamic Epistemic MOdelling.

- DEMO stands for Dynamic Epistemic MOdelling.
- DEMO is a truly *dynamic* epistemic model checker

- DEMO stands for Dynamic Epistemic MOdelling.
- DEMO is a truly *dynamic* epistemic model checker
- DEMO allows for the specification and graphical display of epistemic models *and* action models, and for formula evaluation in epistemic states, including epistemic states specified as (possibly iterated) restricted modal products.

- DEMO stands for Dynamic Epistemic MOdelling.
- DEMO is a truly *dynamic* epistemic model checker
- DEMO allows for the specification and graphical display of epistemic models *and* action models, and for formula evaluation in epistemic states, including epistemic states specified as (possibly iterated) restricted modal products.
- DEMO is written in de programming language Haskell.

- DEMO stands for Dynamic Epistemic MOdelling.
- DEMO is a truly *dynamic* epistemic model checker
- DEMO allows for the specification and graphical display of epistemic models *and* action models, and for formula evaluation in epistemic states, including epistemic states specified as (possibly iterated) restricted modal products.
- DEMO is written in de programming language Haskell.
- Its code implements a reduction of action model logic *AMC* to *PDL*

- DEMO stands for Dynamic Epistemic MOdelling.
- DEMO is a truly *dynamic* epistemic model checker
- DEMO allows for the specification and graphical display of epistemic models *and* action models, and for formula evaluation in epistemic states, including epistemic states specified as (possibly iterated) restricted modal products.
- DEMO is written in de programming language Haskell.
- Its code implements a reduction of action model logic *AMC* to *PDL*
- An introduction in DEMO
- Sum and Product in DEMO

- In $\mathcal{L}_! (A, P)$ the interpretation of an epistemic action is a binary relation between epistemic states that is computed from similar relations but that interpret its subactions

- In $\mathcal{L}_{!}(A, P)$ the interpretation of an epistemic action is a binary relation between epistemic states that is computed from similar relations but that interpret its subactions; therefore we call it a *relational* action language.

- In $\mathcal{L}_{!}(A, P)$ the interpretation of an epistemic action is a binary relation between epistemic states that is computed from similar relations but that interpret its subactions; therefore we call it a *relational* action language.
- In $\mathcal{L}_{KC\otimes}(A, P)$ the description of an epistemic action is a structure that resembles a Kripke model

- In $\mathcal{L}_{!}(A, P)$ the interpretation of an epistemic action is a binary relation between epistemic states that is computed from similar relations but that interpret its subactions; therefore we call it a *relational* action language.
- In $\mathcal{L}_{KC\otimes}(A, P)$ the description of an epistemic action is a structure that resembles a Kripke model; therefore we call it an action model language.

- In $\mathcal{L}_{!}(A, P)$ the interpretation of an epistemic action is a binary relation between epistemic states that is computed from similar relations but that interpret its subactions; therefore we call it a *relational* action language.
- In $\mathcal{L}_{KC\otimes}(A, P)$ the description of an epistemic action is a structure that resembles a Kripke model; therefore we call it an action model language.
- **EA** does not have, until today (25/05/2021) a completeness theorem.

- In $\mathcal{L}_!(A, P)$ the interpretation of an epistemic action is a binary relation between epistemic states that is computed from similar relations but that interpret its subactions; therefore we call it a *relational* action language.
- In $\mathcal{L}_{KC\otimes}(A, P)$ the description of an epistemic action is a structure that resembles a Kripke model; therefore we call it an action model language.
- **EA** does not have, until today (25/05/2021) a completeness theorem.
- **AMC** does have a completeness theorem.

We've described in $\mathcal{L}_!(A, P)$ the action

$$\mathsf{mayread} := L_{ab}\left(L_a?p \cup L_a?\neg p \cup !?\top\right)$$

wherein, Aggela and Baggelis learn that Aggela learns that p, or that Aggela learns that $\neg p$, or that 'nothing happens', and actually nothing happens.

EA→AMC | Buy or sell?

We've described in $\mathcal{L}_!(A, P)$ the action

$$\mathsf{mayread} := L_{ab}\left(L_a?p \cup L_a?\neg p \cup !?\top\right)$$

wherein, Aggela and Baggelis learn that Aggela learns that p, or that Aggela learns that $\neg p$, or that 'nothing happens', and actually nothing happens.
The type of the action mayread is

$$L_{ab}\left(L_a?p \cup L_a?\neg p \cup ?\top\right)$$

and there are three actions of that type, namely,

$$L_{ab}\left(!L_a?p \cup L_a?\neg p \cup ?\top\right)$$

$$L_{ab}\left(L_a?p \cup !L_a?\neg p \cup ?\top\right)$$

$$L_{ab}\left(L_a?p \cup L_a?\neg p \cup !?\top\right)$$

We've described in $\mathcal{L}_!(A,P)$ the action

$$\mathsf{mayread} := L_{ab}\left(L_a?p \cup L_a?\neg p \cup !?\top\right)$$

wherein, Aggela and Baggelis learn that Aggela learns that p, or that Aggela learns that $\neg p$, or that 'nothing happens', and actually nothing happens.
The type of the action $\mathsf{mayread}$ is

$$L_{ab}\left(L_a?p \cup L_a?\neg p \cup ?\top\right)$$

and there are three actions of that type, namely,

$$L_{ab}\left(!L_a?p \cup L_a?\neg p \cup ?\top\right)$$

$$L_{ab}\left(L_a?p \cup !L_a?\neg p \cup ?\top\right)$$

$$L_{ab}\left(L_a?p \cup L_a?\neg p \cup !?\top\right)$$

The 'preconditions' of these actions are, respectively $p, \neg p, \top$.

EA→AMC | Buy or sell?

- Baggelis cannot tell which of those actions actually takes place: they are all the same to him.
- Aggela can distinguish all three actions.

- Baggelis cannot tell which of those actions actually takes place: they are all the same to him.
- Aggela can distinguish all three actions.

This induces a syntactic accessibility among epistemic actions; e.g., that

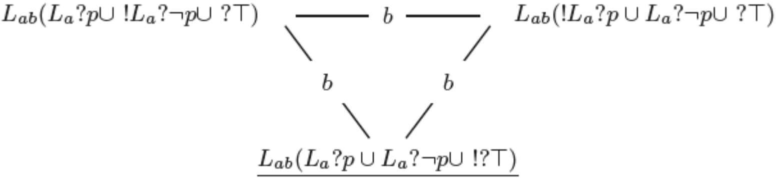
$$L_{ab}\,(!L_a?p \cup L_a?\neg p \cup ?\top) \sim_b L_{ab}\,(L_a?p \cup !L_a?\neg p \cup ?\top)$$

while

$$L_{ab}\,(!L_a?p \cup L_a?\neg p \cup ?\top) \nsim_a L_{ab}\,(L_a?p \cup !L_a?\neg p \cup ?\top)$$

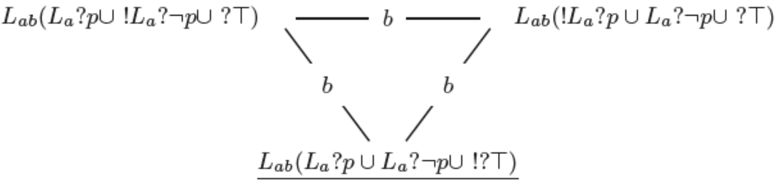
EA→AMC | Buy or sell?

We can visualise this access among the three $\mathcal{L}_!$ actions as

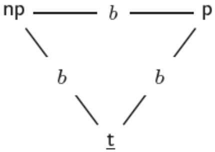


EA→AMC | Buy or sell?

We can visualise this access among the three $\mathcal{L}_!$ actions as



We may replace them by labels p, np and t with preconditions $p, \neg p$, and $\top$, respectively and get action $(\mathsf{Mayread}, \mathsf{t})$

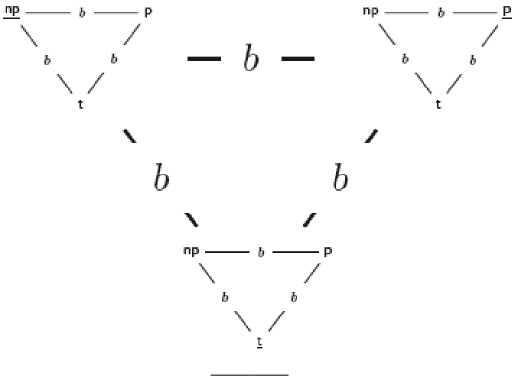


EA→AMC | Buy or sell?

But it is interesting to observe that we might have done a similar trick with the three epistemic actions $(\mathsf{Mayread}, \mathsf{p})$, $(\mathsf{Mayread}, \mathsf{np})$, and $(\mathsf{Mayread}, \mathsf{t})$ by the much simpler expedient of lifting the notion of accessibility between points in a structure to accessibility between pointed structures.

EA→AMC | Buy or sell?

But it is interesting to observe that we might have done a similar trick with the three epistemic actions $(\mathsf{Mayread}, \mathsf{p})$, $(\mathsf{Mayread}, \mathsf{np})$, and $(\mathsf{Mayread}, \mathsf{t})$ by the much simpler expedient of lifting the notion of accessibility between points in a structure to accessibility between pointed structures.



EA→AMC

This method does not apply to arbitrary $\mathcal{L}_!$ actions, because we do not know a notion of syntactic access among $\mathcal{L}_!$ actions that exactly corresponds to the notion of semantic access.

EA←AMC

Vice versa, given an action model, we can construct a $\mathcal{L}_{!\cap}$ action; i.e. the language of epistemic actions *with concurrency*.

EA←AMC

Vice versa, given an action model, we can construct a $\mathcal{L}_{!\cap}$ action; i.e. the language of epistemic actions *with concurrency*.

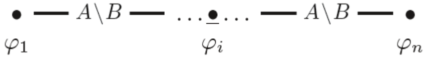
Interestingly, there has been (independently) given a completeness theorem for this logic! (see also)

Consider the case where a subgroup B of all agents A is told which of n alternatives described by propositions $\varphi_1, \ldots, \varphi_n$ is actually the case, but such that the remaining agents do not know which from these alternatives that is. Let φ_i be the actually told proposition.

EA←AMC | Example 6.40

Consider the case where a subgroup B of all agents A is told which of n alternatives described by propositions $\varphi_1, \ldots, \varphi_n$ is actually the case, but such that the remaining agents do not know which from these alternatives that is. Let φ_i be the actually told proposition.

In $\mathcal{L}_{KC\otimes}$ this is described as a pointed action model visualised as (with the preconditions below the unnamed action points)



EA←AMC | Example 6.40

Consider the case where a subgroup B of all agents A is told which of n alternatives described by propositions $\varphi_1, \ldots, \varphi_n$ is actually the case, but such that the remaining agents do not know which from these alternatives that is. Let φ_i be the actually told proposition.

In $\mathcal{L}_{KC\otimes}$ this is described as a pointed action model visualised as (with the preconditions below the unnamed action points)

In $\mathcal{L}_!$ the coresponding epistemic action is

$$L_A\,(L_B?\varphi_1 \cup \cdots \cup !L_B\varphi_i \cup \cdots \cup L_B\varphi_n)$$

In this section we pay attention to modelling private (truthful) announcements, that transform an epistemic state into a *belief* state, where agents not involved in private announcements lose their access to the actual world.

In this section we pay attention to modelling private (truthful) announcements, that transform an epistemic state into a *belief* state, where agents not involved in private announcements lose their access to the actual world.

In different words: agents that are unaware of the private announcement therefore have false beliefs about the actual state of the world, namely, they believe that what they knew before the action, is still true.

The proper general notion of action model is as follows

Definition (Action model for belief)

Let $\mathcal{L}$ be a logical language for given parameters agents A and atoms P. An action model M is a structure $\langle \mathsf{S}, \mathsf{R}, \mathsf{pre} \rangle$ s.t. S is a domain of *action points*, s.t. for each $a \in A$, R_a is an accessibility relation on S, and s.t. $\mathsf{pre} : \mathsf{S} \rightarrow \mathcal{L}$ is a preconditions function that assigns a *precondition* $\mathsf{pre}(\mathsf{s}) \in \mathcal{L}$ to each $\mathsf{s} \in \mathsf{S}$. A *pointed action model* is a structure (M, s) with $\mathsf{s} \in \mathsf{S}$.

One also has to adjust various other definitions, namely those of action model language, action model execution, and action model composition.

The ‘typical’ action that needs such a more general action model is the ‘private announcement to a subgroup’ mentioned above.

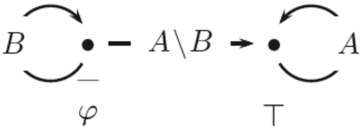
The ‘typical’ action that needs such a more general action model is the ‘private announcement to a subgroup’ mentioned above.

Let subgroup B of the public A learn that φ is true, without the remaining agents realising (or even suspecting) that.

The 'typical' action that needs such a more general action model is the 'private announcement to a subgroup' mentioned above.

Let subgroup B of the public A learn that φ is true, without the remaining agents realising (or even suspecting) that.

The action model for that is pictured below



Example 6.43

Consider the epistemic state $(Letter, 1)$ where Aggela and Baggelis are uncertain about the truth of p.

Example 6.43

Consider the epistemic state $(\mathit{Letter}, 1)$ where Aggela and Baggelis are uncertain about the truth of p.

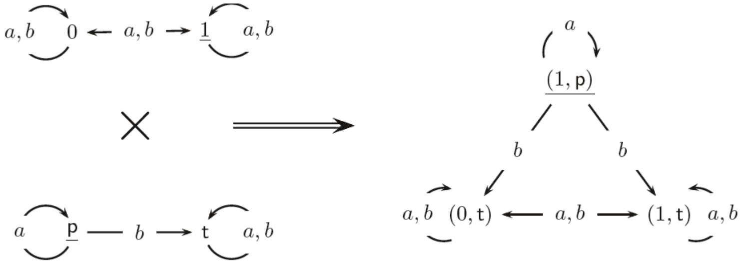
The epistemic action that Aggela learns p without Baggelis noticing that, consists of two action points p and t, with preconditions p and $\top$, and p actually happens.

Example 6.43

Consider the epistemic state $(Letter, 1)$ where Aggela and Baggelis are uncertain about the truth of p.

The epistemic action that Aggela learns p without Baggelis noticing that, consists of two action points p and t, with preconditions p and $\top$, and p actually happens.

The model and its execution are pictured below.



Example 6.43+

Model and execute the action where Aggela secretly reads the letter and learns p, while thinking that Baggelis doesn't see her, but Baggelis does see her reading the letter, without learning the content of the letter.

Example 6.43+

Model and execute the action where Aggela secretly reads the letter and learns p, while thinking that Baggelis doesn't see her, but Baggelis does see her reading the letter, without learning the content of the letter.

Thus, in the final epistemic model, Aggela knows that p holds, Baggelis doesn't know that p, but he knows, that Aggela knows whether p or $\neg p$, and Aggela believes that Baggelis doesn't know that Aggela knows whether p or $\neg p$;
i.e.

$$K_a p$$

$$\neg (K_b p \vee K_b \neg p)$$

$$K_b (K_a p \vee K_a \neg p)$$

$$\neg K_a (K_b (K_a p \vee K_a \neg p))$$

Example 6.43+

We model this action in two steps.

Example 6.43+

We model this action in two steps.

- Firstly, we assume that a private announcement is being made in which Aggela learns whether p or $\neg p$, and she actually learns p.

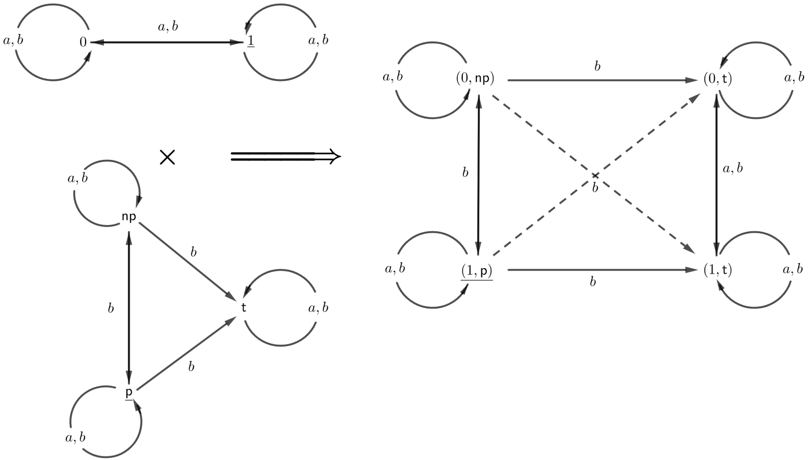
Example 6.43+

We model this action in two steps.

- Firstly, we assume that a private announcement is being made in which Aggela learns whether p or $\neg p$, and she actually learns p.
- Secondly, we assume that a private announcement is being made in which Baggelis learns that Aggela knows whether p or $\neg p$, and she actually learns p.

Example 6.43+ | 1st announcement

The preconditions of $\mathsf{np}, \mathsf{p}, \mathsf{t}$ are defined as usual.



Example 6.43+ | 2nd announcement

The precondition of k is $\mathsf{pre}(\mathsf{k}) := K_a p \vee K_a \neg p$ and the precondition of t' is $\mathsf{pre}(\mathsf{t}') := \top$.